Measurement of the CKM matrix element $|V_{ub}|$ studying Exclusive Semileptonic Decays on the Recoil of Fully Reconstructed $B$ ’s with the \textit{BABAR} detector

Tesi presentata da

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Introduction

The main physics goal of the BABAR experiment is to establish CP violation in B meson decays and to test whether the observed effects are compatible with the predictions of the Standard Model. CP violating effects result in the SM from an irreducible phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the couplings of the charged weak currents to quarks. A precise determination of the absolute value of the CKM matrix element $|V_{ub}|$, the coupling strength of the $b$ quark to the $u$ quark, will significantly enhance the constraints of the unitarity of the CKM matrix and thus to test the consistency with the SM.

The measurement of $|V_{ub}|$ requires the study of a $b \to u$ transition in a well-understood context. Semileptonic $b \to u \ell \nu$ decays are best for that purpose since they are much easier to understand than hadronic decays from a theoretical point of view, and much easier to study experimentally than purely leptonic decays because they are far more abundant.

The extraction of $|V_{ub}|$ is a challenge both for theory and experiment. Experimentally, the main problem is the separation of $b \to u \ell \nu$ decays from the dominant $b \to c \ell \nu$ decays. The amplitude for these two processes are proportional to $|V_{ub}|$ and $|V_{cb}|$, respectively. The relative rates (at the quark level) are roughly

$$\frac{\Gamma(b \to u \ell \nu)}{\Gamma(b \to c \ell \nu)} \approx 2\frac{|V_{ub}|^2}{|V_{cb}|^2} \approx 0.02,$$

(0.1)

where $|V_{ub}/V_{cb}| = 0.1$ has been assumed, and the numerical factor of $\approx 2$ accounts for the difference in phase space. The relative rate for the inclusive decays differs by a factor of 50. However, relative to single exclusive decays like $B \to \pi \ell \nu$, the total inclusive semileptonic rate is a factor of 500 larger. The need to suppress the dominant $B \to X_c \ell \nu$ decays leads to a set of selection criteria that restrict the phase space. This generally makes the theoretical extrapolation to the full decay rate more difficult.

Most of the earlier measurements of $|V_{ub}|$ have been derived from inclusive distributions, exploiting the kinematic differences between charm and charmless semileptonic decays. The inclusive semileptonic rates can be reliably calculated from theory at the parton level and the current precision achieved by inclusive measurements is around 8%.

Alternatively $|V_{ub}|$ can be measured by exclusive charmless semileptonic decays. The study of exclusive decays offers some experimental advantages compared to an inclusive study, such as the possibility of keeping a higher fraction of the phase space and permitting a better background rejection. On the other hand, it deals with lower statistics and it is affected by large theoretical uncertainties arising from the calculation of the form factors describing the strong interaction effects on the hadronization of the different $X_c$ final states. A present issue in the exclusive determination of $|V_{ub}|$ is the fact that the measured values lie quite below the inclusive average and this could be related to the poorly known dynamics of such decays.

In the high luminosity scenario provided by the $B$-Factories a better estimation of exclusive decays is now possible and a competitive exclusive $|V_{ub}|$ measurement can be obtained.
At the $B$-Factories $B \bar{B}$ events are produced at $\Upsilon(4S)$ resonance. The analysis described in this thesis exploits events with a reconstructed $B$ decay to hadronic final states, which make up the optimal environment for the study of semileptonic decays of the second $B$ meson. The relative small background allows for loose selection criteria and reduces the uncertainty in the extrapolation to the full decay rate. Due to the full reconstruction of one of the two $B$’s, the kinematics of the event is constrained and the missing mass of the event originate only from the undetected neutrino.

Based on a sample of $315 \, fb^{-1}$ of $B \bar{B}$ events collected by the $B\bar{B}$AR experiment at the PEP-II asymmetric-energy $B$-Factory at SLAC, a study of the exclusive charmless semileptonic $B$ decays $B^0 \rightarrow \pi^- \ell^+\nu$, $B^+ \rightarrow \pi^0 \ell^+\nu$, $B^+ \rightarrow \eta\ell^+\nu$ and $B^+ \rightarrow \eta'\ell^+\nu$ is presented. In particular, the $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ branching fractions is performed in bins of the invariant mass of the leptonic system $q^2 = (p_\ell + p_\nu)^2$, and these partial branching fractions are used to determine $V_{ub}$ using different theoretical calculation of the form factors, that are reliable only in different limited $q^2$ ranges. The $|V_{ub}|$ measurement is extracted also from the total branching fraction using as theoretical input the extrapolation of the form factors to the full $q^2$ range.

In the first chapter the theoretical framework needed for $|V_{ub}|$ extraction from exclusive charmless semileptonic $B$ meson decays, the different theoretical models for the calculation of the form factors and the different experimental approaches used to study charmless semileptonic $B$ decays are briefly described. The second chapter is a description of the PEP-II $e^+ e^-$ asymmetric collider and the $B\bar{B}$AR detector. The first half of the third chapter is an overview of the particle identification, tracks and neutrals reconstruction and selection relevant to this analysis. In the second half of the chapter the reconstruction of a $B$ in a fully hadronic decay is detailed. The studies on the charmless semileptonic decays start from the fourth chapter, in which the selection criteria applied to isolate the $B \rightarrow X_u \ell\nu$ exclusive channels are described. The technique used for the extraction of the charmless total and partial branching fractions is described in the fifth chapter. The sixth chapter is devoted to systematic studies. And finally in the last chapter the final results and the extraction of $|V_{ub}|$ are presented and discussed. Improvements on the selection of the semileptonic decays and on the technique used to extract the branching fractions has been introduced by the candidate. The innovative part of the analysis is the measurement of the $B \rightarrow \pi\ell\nu$ partial branching fraction in bins of $q^2$ and the related extraction of $|V_{ub}|$. Until now, in fact, the statistics of these measurements using the hadronic recoil technique was too small to allow binning in $q^2$.
CKM Matrix and Semileptonic B decays

1.1 Standard Model and CKM Matrix

The electroweak sector of the Standard Model (SM) [1] is governed by a gauge theory based on the local group $SU_L(2) \otimes U_Y(1)$, which describes the symmetries of the matter fields. The Yang-Mills electroweak Lagrangian is:

$$\mathcal{L} = -\frac{1}{4} \sum_A W^A_{\mu\nu} W^{A\mu\nu} - \frac{1}{2} B_{\mu\nu} B^{\mu\nu} + \bar{\Psi}_L i \gamma^\mu D_\mu \Psi_L + \bar{\Psi}_R i \gamma^\mu D_\mu \Psi_R,$$

(1.1)

where, the spinors $\Psi_L$ and $\Psi_R$ represent the matter fields in their chiral components, and the field strength tensors are given by:

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g \epsilon_{ABC} W^B_\mu W^C_\nu \quad \text{and} \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

(1.2)

Here $W$ and $B$ are the SU(2) and U(1) gauge fields, with the coupling constants $g$ and $g'$, and $\epsilon_{ABC}$ is the totally anti-symmetric Levi-Civita tensor. The corresponding covariant derivative is:

$$D_\mu \Psi_{L,R} = \left[ \partial_\mu + ig \Sigma^A_{L,R} W^A_\mu + ig' \frac{1}{2} Y_{L,R} B_\mu \right] \Psi_{L,R},$$

(1.3)

where $t^A_{L,R}$ and $\frac{1}{2} Y_{L,R}$ are the SU(2) (weak isospin) and U(1) (hypercharge) generators. The electric charge generator is related to the isospin and hypercharge by:

$$Q = t^3_L + \frac{1}{2} Y_L = t^3_R + \frac{1}{2} Y_R.$$  

(1.4)

The left and the right fermion components have different properties under the gauge group. The left components behave as doublets while the right as singlets. In the symmetric limit the two chiral component cannot interact each other, and thus mass term for fermions (of the form $\bar{\Psi}_L \Psi_R$) are forbidden. To give mass terms to fermions as well as to gauge bosons, the electroweak theory is realized with a vacuum state only invariant under the $U_{EM}(1)$ electric charge gauge transformation (spontaneous symmetry breaking). The gauge theories spontaneous broken allow to introduce mass terms for the gauge boson and the fermion fields without spoiling the gauge invariance, and the renormalizability of the theory. The mechanism by which, starting from a degenerate vacuum state, mass terms are introduced is known as Higgs mechanism [2]. The Higgs Lagrangian term is:

$$\mathcal{L}_{Higgs} = (D_m \phi) \dagger (D^\mu \phi) - V(\phi \dagger \phi) - \bar{\Psi}_L \Gamma \Psi_R \phi - \bar{\Psi}_R \Gamma^\dagger \Psi_L \phi,$$

(1.5)
where $\phi$ is the isospin doublet of the Higgs scalar fields and the quantities $\Gamma$ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The general form of the Higgs potential is:

$$V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2,$$ 

(1.6)

and it is not possible to include terms with higher dimensions without breaking the renormalizability of the SM. To have a vacuum state (the minimum of the potential) degenerate, the $\mu^2$ coefficient should be negative, while the coefficient $\lambda$ should be positive to guarantee the potential bound from below. Under these hypotheses, the vacuum state of the Higgs field satisfies $|\phi^2| = -\mu^2/2\lambda = v^2$. The field $\phi$ can be expanded around one of its ground states; in choosing a particular ground state ($\phi_0 = \left( \begin{array}{c} 0 \\ v \end{array} \right)$), the $SU_{L,R}(2) \otimes U_Y(1)$ symmetry is spontaneously broken.

The mass terms for the gauge bosons are coming from the kinetic part of the Higgs Lagrangian once it is expanded around the Higgs vacuum state. The correct quantum numbers of the Higgs field are fixed by the requirement that the Lagrangian, defined by Eq.(1.5), is gauge invariant.

<table>
<thead>
<tr>
<th>Family</th>
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<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\left( \begin{array}{c} \nu_e \ e \end{array} \right)_L$</td>
<td>$\left( \begin{array}{c} \nu_\mu \ \mu \end{array} \right)_L$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>$\mu_R$</td>
</tr>
<tr>
<td>$\left( \begin{array}{c} u \ d \end{array} \right)_L$</td>
<td>$\left( \begin{array}{c} c \ s \end{array} \right)_L$</td>
</tr>
<tr>
<td>$u_R$</td>
<td>$c_R$</td>
</tr>
<tr>
<td>$d_R$</td>
<td>$s_R$</td>
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Table 1-1. Electroweak interaction multiplets. $t$ indicates the isospin, $t_3$ is the third isospin component, $Y$ indicates the hyper-charge and $Q$ indicates the electric charge.

Since the $SU_L(2) \otimes U_Y(1)$ symmetry is spontaneously broken into $U_{EM}(1)$, only the linear combination of gauge fields with the quantum numbers of the photon remains massless. A general linear combination

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between the gauge bosons associated to the generator in Eq.(1.4) can be written:

\[
\begin{pmatrix}
A_\mu \\
Z_\mu
\end{pmatrix} =
\begin{pmatrix}
-\sin \theta_W & \cos \theta_W \\
\cos \theta_W & \sin \theta_W
\end{pmatrix}
\begin{pmatrix}
W^3_\mu \\
B_\mu
\end{pmatrix}.
\]

(1.7)

where the angle $\theta_W$ is known as the Weak or Weinberg mixing angle. Once the symmetry is spontaneously broken through the interaction with the Higgs field, $A_\mu$ remains massless while $Z_\mu$, $W^+_\mu$ and $W^-_\mu$ acquire a mass term. $W^+_\mu$ and $W^-_\mu$ are defined as:

\[
W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm iW^2_\mu).
\]

(1.8)

The bilinear terms in the fields $Z_\mu$ and $W^\pm_\mu$ in Eq. (1.5) can be identified as the mass terms:

\[
M^2_Z = \frac{v^2 g^2}{2 \cos^2 \theta_W}
\]

\[
M^2_W = \cos^2 \theta_W M^2_Z
\]

(1.9)

(1.10)

which implies $\tan \theta_W = g'/g Y_\phi$. In terms of these new fields the fermionic currents are:

\[
J^\pm_\mu = \Sigma_f \bar{\psi}^f_j (1 - \gamma_5) \gamma_\mu t^\pm \psi^f_j
\]

(1.11)

\[
J^0_\mu = \Sigma_f \bar{\psi}^f_j \gamma_\mu \left[(1 - \gamma_5)t^3 - 2Q \sin^2 \theta_W\right] \psi^f_j,
\]

(1.12)

\[
J^{em}_\mu = \Sigma_f \bar{\psi}^f_j \gamma_\mu Q \psi^f_j,
\]

(1.13)

where $\bar{\psi}^f_j$ represents the isospin doublet for the fermions fields (see Tab.1-1) with $f$ acting as a family index, $(1 - \gamma_5)$ is the left-handed chiral projector, and $t^\pm$ are the isospin generator associated to the fields $W^\pm_\mu$. The first current describes interactions which change the electric charge, while the other two, produce transitions charge-conserving. The Lagrangian (1.1) could be rewritten in two terms: one including interactions between the neutral current and the $A_\mu$ and $Z_\mu$ bosons, and another describing the interactions of the $W^\pm_\mu$ with the charged current:

\[
\mathcal{L}_D = \mathcal{L}_{CC} + \mathcal{L}_{NC},
\]

(1.14)

\[
\mathcal{L}_{CC} = \frac{g_2}{2 \sqrt{2}}(J^+_\mu W^+_\mu + J^-_\mu W^-_\mu),
\]

(1.15)

\[
\mathcal{L}_{NC} = -e J^{em}_\mu A_\mu + \frac{g_2}{2 \cos \theta_W} J^0_\mu Z^\mu,
\]

(1.16)

where $e$ is defined as $e = g_2 \sin \theta_W$.

Starting from the same doublet which gives masses to the gauge bosons it is possible to introduce mass terms for the fermion fields. This imposes others restrictions on the Higgs field. To obtain fermion mass terms like:

\[
-\bar{\psi}^f_L \Gamma \psi^f_R \phi - \bar{\psi}^f_R \Gamma \psi^f_L \bar{\phi} \quad \text{where} \quad \bar{\phi} = i \sigma^2 \phi^\dagger,
\]

(1.17)

invariant under $SU_{L,R}(2)$ transformations, the Higgs field is required to have isospin equal to 1/2. The $\Gamma$ matrices contain the Yukawa constants, which determine the strength of the fermion couplings to the Higgs fields.

**CKM Matrix and Semileptonic B Decays**
The fermion mass matrix is obtained from the Yukawa couplings expanding $\phi$ around the vacuum state:

$$M = \bar{\psi}_L \mathcal{M} \psi_R + \bar{\psi}_R \mathcal{M}^\dagger \psi_L \ ,$$

(1.18)

with

$$\mathcal{M} = \Gamma \cdot v \ .$$

(1.19)

It is important to observe that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, $\gamma_5$-free, and diagonal. In fact, we can make separate unitary transformations on $\psi_L$ and $\psi_R$ according to

$$\psi'_L = L \psi_L, \quad \psi'_R = R \psi_R,$$

(1.20)

and consequently

$$\mathcal{M} \rightarrow \mathcal{M}' = L^\dagger \mathcal{M} R \ .$$

(1.21)

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}$.

Weak charged currents are the only tree level interactions in the SM that may induce a change of flavor. By emission of a $W$ boson an up-type quark is turned into a down-type quark, or a $\nu_\ell$ neutrino is turned into a $\ell^-$ charged lepton. If we start from an up quark that is a mass eigenstate, emission of a $W$ turns it into a down-type quark state $d'$ (the weak isospin partner of $u$) that in general is not a mass eigenstate. In general, the mass eigenstates and the weak eigenstates do not in fact coincide and a unitary transformation connects the two sets:

$$\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix} = V
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix},$$

(1.22)

where $V$ is the Cabibbo-Kobayashi-Maskawa matrix (CKM)[3]. Thus in terms of mass eigenstates the charged weak current of quarks has the form:

$$J_\mu^+ \propto \bar{u}_\mu (1 - \gamma_5) t^+ V d_i.$$

(1.23)

Since $V$ is unitary (i.e. $VV^\dagger = V^\dagger V = 1$) and commutes with $T^2$, $T_3$ and $Q$ (because all d-type quarks have the same isospin and charge) the neutral current couplings are diagonal both in the primed and unprimed basis. If the $Z$ down-type quark current is abbreviated as $\bar{d'} \Gamma d'$ then, by changing basis we get $\bar{d'} \Gamma V d$ and $V$ and $\Gamma$ commute; it follows that $\bar{d'} \Gamma d' = \bar{d} \Gamma d$. This is the Glashow - Iliopoulos - Maiani (GIM) mechanism [4] that ensures natural flavor conservation of the neutral current couplings at the tree level.

With three fermion generations the matrix $V$ could be expressed in terms of three angles and one irremovable complex phase [5]. The CKM matrix is usually represented as:

$$V = \begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \ .$$

(1.24)

The irremovable phase in the CKM matrix allows possible $CP$ violation.

The measurement of the elements of the CKM matrix is fundamental to test the validity of the SM. Many of them (actually the first two rows of the matrix) are measured directly, namely by tree-level processes.

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unitary relations one can put constraints on the top mixing $|V_{ts}|$. Moreover the $B$ mixing measurements, that involve box diagrams, can give information also about $V_{td}$ and $V_{tb}$.

The CKM-matrix can be expressed in terms of four Wolfenstein parameters $(\lambda, A, \rho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and $\eta$ representing the $CP$ -violating phase [6]:

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \quad (1.25)$$

$\lambda$ is small, and for each element in $V$, the expansion parameter is actually $\lambda^2$.

The Wolfenstein parameterization offers a transparent geometrical representation of the structure of the CKM matrix. The unitarity of the matrix implies various relations among its elements. Three of them are very useful for understanding the SM predictions for $CP$ violation:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad (1.26)$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad (1.27)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (1.28)$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are “the Unitarity Triangles”. If the $CP$ symmetry is violated the area of the triangles is not zero. The $B$ physics is related to the third triangle at least for what the $B$ Factory can access. The study of the parameters of this triangle encompasses the physics of $CP$ violation in the SM. The openness of this triangle, due to the fact that all the three sides are of the same order of magnitude, predicts large $CP$ asymmetries.

It should be remarked that the Wolfenstein parameterization is an approximation and neglecting $O(\lambda^4)$ terms could be wrong in particular processes. An improved approximated parameterization of the original Wolfenstein matrix is given in [7]. Defining

$$V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ub} = A\lambda^3(\rho - i\eta), \quad (1.29)$$

one can then write

$$V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta}), \quad (1.30)$$

$$\text{Im}(V_{cd}) = -A^2\lambda^5\eta, \quad \text{Im}(V_{ts}) = -A\lambda^4\eta, \quad (1.31)$$

where

$$\bar{\rho} = \rho(1 - \lambda^2/2), \quad \bar{\eta} = \eta(1 - \lambda^2/2), \quad (1.32)$$

turn out to be excellent approximations to the exact expressions. Depicting the rescaled Unitarity Triangle in the $(\bar{\rho}, \bar{\eta})$ plane, the lengths of the two complex sides are

$$R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{1 - \lambda^2/2}{\lambda} \frac{|V_{ub}|}{|V_{cb}|}, \quad (1.33)$$

CKM MATRIX AND SEMILEPTONIC B DECAYS
Figure 1-1. The rescaled Unitarity Triangle, all sides divided by $V_{cb}^*V_{cd}$.

\[ R_t \equiv \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|. \]  

(1.34)

The rescaled Unitarity Triangle (Fig. 1-1) is derived from Eq.(1.28) by choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, dividing the lengths of all sides by $V_{cd}V_{cb}^*$, aligns one side of the triangle with the real axis and makes the length of this side equal to 1.

The form of the triangle is unchanged. Two vertices of the rescaled Unitarity Triangle are thus fixed at $(0,0)$ and $(1,0)$. The coordinates of the remaining vertex are denoted by $(\bar{\rho}, \bar{\eta})$. Both angles and sides can be measured in a $B$ factory and they can offer two independent tests of the Standard Model. Inconsistencies between these two tests would indicate the presence of New Physics. The constraints on the apex of the Unitarity Triangle, obtained from different measurements, now overlap in one small area in the first quadrant in the $(\bar{\rho}, \bar{\eta})$ plane [8, 9]. The constraints from the lengths of the sides and independently those from CP violating processes indicate the consistent regions on the $(\bar{\rho}, \bar{\eta})$ plane. The $|V_{ub}|/|V_{cb}|$ constraint shown in Fig.1-2 can be directly derived from Eq.(1.33). The CKM elements $|V_{ub}|$ and $|V_{cb}|$ therefore provide a test of the SM by over-constraining the $(\bar{\rho}, \bar{\eta})$ plane vertex with other measurements. These elements can be directly determined from $b \to u\ell\nu$ and $b \to c\ell\nu$ decays respectively. While $|V_{cb}|$ has been measured with a 2% uncertainty [10], $|V_{ub}|$ still remains one of the least known elements of the CKM matrix and dominates the error on the length of the side opposite to the angle $\beta$.

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1.2 Semileptonic B decays

Semileptonic decays provide an excellent tool in which it is possible to measure $|V_{ub}|$ and $|V_{cb}|$. These processes also allow the study of the effects of non-perturbative QCD interactions on weak decays. The strong interactions are difficult to be calculated but, in semileptonic decays, they are isolated to the hadronic current. As a consequence the effect of the strong interactions can be rigorously parametrized in terms of a small number of form factors, which are functions of the Lorentz-invariant quantity $q^2$, the square of the mass of the virtual W boson.

The Unitarity Triangle analysis shows the impressive success of the CKM picture in describing $CP$ violation in the SM, but, with the increasing precision of the experimental results, a slight disagreement between $\sin 2\beta$ and $|V_{ub}|$ is appeared in the UT fit, as shown in Fig.1-2. This disagreement could be due to some problem with theoretical calculation and/or with the estimate of the uncertainties on the $|V_{ub}|$ measurements. So an effort must be done for a substantial improvement of the theoretical and experimental accuracy for this quantity. In the future, if the $|V_{ub}|$ value will be confirmed by more precise data and theory calculations, the disagreement in the UT fit might reveal a bound of New Physics phase in $B_d$ mixing [8].

**Figure 1-2.** Unitarity Triangle constrains from different measurements of the Standard Model parameters in the $(\rho, \theta)$ plane, updated to the Summer 2006 results [8]. Allowed region for $(\rho, \theta)$, using all available measurements, is shown with closed contours at 68% and 95% probability. The full lines correspond to 95% probability regions for the constraints, given by the measurements of $|V_{ub}|/|V_{cb}|$, $\epsilon_K$, $\Delta m_d/\Delta m_s$, $\alpha$, $\beta$, $\gamma$, $\Delta \Gamma_d/\Gamma_d$, $\Delta \Gamma_s/\Gamma_s$, $A_{S_L}^d$, and the dimuon asymmetry from $D^0$. 
For $b \to c \ell \nu$ decays, the large masses of both the $b$ and $c$ quarks provide the key to reliable theoretical predictions based on the Heavy-Quark Effective Theory (HQET) for exclusive decays [11, 12] and Heavy-Quark Expansion (HQE) for inclusive decays [13]. In the Heavy-Quark Symmetry limit ($m_b$ and $m_c \to \infty$) the hadronic system is unchanged by replacing one heavy quark by the other one. Since the $b$ and $c$ quark masses are not truly infinite, there are corrections to these predictions but they are relatively small. So the Heavy-Quark Symmetry relates form factors to each other, reducing the number of independent functions and gives the normalization at zero-recoil configurations, that is when the daughter $c$ hadron has zero momentum with respect to the parent $b$ hadron.

On the other hand, for $b \to u \ell \nu$ decays, due to the small values of the $u$ quark mass compared to the scale of the energy transfer, the zero-recoil configuration does not provide a solid normalization point. While the determination of $|V_{ub}|$ is improving, both experimentally and theoretically, there are still large uncertainties and significant challenges in understanding the $b \to u \ell \nu$ semileptonic decays. The problem with the exclusive $b \to u \ell \nu$ decays is that the strong hadronic dynamics cannot be calculated from first principles and it has to resort to models, such as Light-Cone Sum Rules (LCSR) or Lattice QCD (LQCD) calculations, to obtain the form factor.

1.3 Exclusive semileptonic B decays and determination of $|V_{ub}|$

Exclusive $B \to X_u \ell \nu$ decays proceed, at the tree level, via a $b \to uW$ transition with a spectator quark, as schematically represented by the Feynman diagram given in Fig.1-3.

![Feynman diagram for the charmless semileptonic decays $B \to X_u \ell \nu$.](image)

The transition amplitude, at tree level, factorizes into leptonic and hadronic parts and the hadronic matrix elements contain the non-perturbative strong interaction effects. Higher order electroweak corrections are suppressed as power of $m_\ell/M_W$ and may be ignored. For processes where momenta are consistently small compared to $M_W$ (i.e. for processes where the $W$ is virtual) the $W$ propagator may be approximated as $G_F/\sqrt{2} = g^2/8M_W^2$. In this approximation, the semileptonic $B$ decay amplitude may be written as [14]:

\[ M(B \to X_u \ell \nu) = \frac{g^2}{8M_W^2} V_{ub} C^\mu H_\mu \]

\[ \text{(1.35)} \]

\[ M(B \to X_u \ell \nu) = \frac{G_F}{\sqrt{2}} V_{ub} C^\mu H_\mu \]

\[ \text{(1.36)} \]
where $G_F$ is the weak interaction’s Fermi constant and the leptonic $\mathcal{L}_\mu$ and hadronic $\mathcal{H}_\mu$ currents are given by:

$$\mathcal{L}_\mu = \bar{u}_\ell \gamma_\mu (1 - \gamma_5) v, \quad (1.37)$$

$$\mathcal{H}_\mu = \langle X_{uq}\rangle J_{\text{had}}^\mu (0) | B_{0q} \rangle, \quad (1.38)$$

The leptonic current is well-known and can be calculated precisely using perturbation theory. The hadronic current is not easily calculable because of the complexity of the low-energy QCD processes inside the mesons. It is customary to introduce form factors to describe the overlap integral of the initial $l$ meson with the final state meson. The form factors are functions of the $q^2 = m_W^2$, the virtual $W$ mass,

$$q^2 = m_W^2 = (p_l + p_\ell)^2 = (p_B - p_X)^2 = M_B^2 + m_{Xu}^2 - 2M_B E_X u, \quad (1.39)$$

where $p_B$ is the energy-momentum four-vector of the $B$ meson, $M_B$ its mass, and $m_{Xu}$ and $E_X$ are the mass and energy of the final meson $X_u$ in the $B$ meson rest frame.

In terms of form factors, the hadronic current is given by a different expression depending on whether the $B$ meson decays either to a pseudo-scalar ($\pi, \eta, \eta'$) or to a vector ($\rho, \omega$) meson final state.

Since in this thesis is presented only the study of the $B \to (\pi^0, \pi, \eta, \eta') l\ell \nu$ decays, we discuss only the $B$ semileptonic decays to a pseudo-scalar ($\pi, \eta, \eta'$) $l\ell \nu$. A discussion of the $B$ semileptonic decays to vector meson is given elsewhere [12, 15].

### 1.3.1 Semileptonic B decays to pseudo-scalar mesons

The form factor calculation is focused on the $B \to \pi l\ell \nu$ decay, since it is the simplest $B$ charmless semileptonic decay mode, both in terms of theoretical prediction and of experimental feasibility. In this section we describe in detail the $B \to \pi l\ell \nu$ decay mode, but most of the problems which exist in this decay are common to other more complicated channels.

For the $B \to \pi l\ell \nu$ decay mode the hadronic current can be written in terms of the form factors $f^+(q^2)$ and $f^0(q^2)$ [16]:

$$\langle \pi(k)| \bar{u}_\ell \gamma_\mu u(p) | B(p) \rangle = f^+(q^2) \left[ (p_B + k_\pi)^\mu - \frac{M_B^2 - m_\pi^2}{q^2} q^\mu \right] + f^0(q^2) \frac{M_B^2 - m_\pi^2}{q^2} q^\mu, \quad (1.40)$$

where $p_B$ is the momentum of the $B$ meson, $k_\pi$ is the momentum of the final state pseudo-scalar meson $\pi$ and $q^2$ is the invariant mass of the virtual $W$ boson. This expression can be simplified for leptons with small masses such as electrons and muons, because in the limit of $m_l \ll M_B$, we have $q^4 L_\mu = 0$. The terms proportional to $q^4$ can be neglected and we are left with the first term and only one form factor $f^+(q^2)$.

Under these assumptions, the differential decay rate can be written as [16]

$$\frac{d\Gamma(B^0 \to \pi^- l^+ \nu)}{dq^2 d\cos \theta_{Wl}} = |V_{ub}|^2 \frac{G_F^2 \alpha^3}{2\pi^3} \sin^2 \theta_W |f^+(q^2)|^2, \quad (1.41)$$

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where the momentum \( k_\pi \) in the center of mass of the \( B \) meson is given by:

\[
|k_\pi| = \sqrt{\left( \frac{M_B^2 + m_\pi^2 - q^2}{2M_B} \right)^2 - m_\pi^2}
\]  
(1.42)

and \( q^2 \) can vary from zero to \( q_{\text{max}}^2 = (M_B - m_\pi)^2 \). The rate is dependent on \( \sin^2 \theta_W \), where \( \theta_W \) is the angle of the charged lepton momentum in the \( W \) rest frame with respect to the \( W \) momentum in the \( B \) rest frame. Integrating Eq.(1.41) over the whole \( \theta_W \) the differential rate is given by the following expression:

\[
\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = 2 \times \frac{d\Gamma(B^+ \to \pi^0 \ell^+ \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |f_+(q^2)|^2 p_\pi^3.
\]  
(1.43)

Combining the theoretical prediction on the form factors and experimental results for the \( B \to \pi \ell \nu \) branching fraction, Eq.(1.43) can be used to extract an estimate for \( |V_{ub}| \):

\[
|V_{ub}| = \sqrt{\frac{\Delta BR(B^0 \to \pi^- \ell^+ \nu)}{\Delta \zeta \cdot \tau_B}},
\]  
(1.44)

where \( \tau_B \) is the \( B \) meson lifetime, \( \Delta BR(B^0 \to \pi^- \ell^+ \nu) \) is the partial branching ratio for a given \( q^2 \) range and \( \Delta \zeta \) is the predicted form-factor normalization over the same \( q^2 \) range and it is defined as:

\[
\Delta \zeta = \frac{G_F^2}{24\pi^3} \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} |f_+(q^2)|^2 p_\pi^3 dq^2.
\]  
(1.45)

### 1.4 Theoretical model for the form factor calculations

As we discussed in previous section, the form factor determination is crucial to extract \( |V_{ub}| \). The form factors can be extracted with different techniques:

- **Quark model, ISGW2** [17],
- **Light Cone Sum Rule (LCSR)** [15, 18, 19, 20],
- **Lattice calculation** [21, 22, 23, 24, 25].

These models use the calculation at a given \( q^2 \) as normalization point. In general, \( q_{\text{max}}^2 \) is used since it represents the point where the calculations are more reliable.

#### 1.4.1 Quark models (ISGW2)

Quark model calculations are used to calculate semileptonic decay form factors by postulating forms for the meson wave functions and using these estimated wave functions to calculate the matrix elements of...
the hadronic currents. Typically these matrix elements are calculated at one extreme values $q^2 = 0$ or $q^2 = q^2_{\text{max}}$. The actual $q^2$ dependence of the form factor is then determined separately, usually by using some simple phenomenological approach. The ISGW model, along with its update ISGW2 [17] is one such constituent quark model with relativistic corrections and it is the only such model considered in this analysis. Calculations are normalized at $q^2 = q^2_{\text{max}}$ and the extrapolation of the form factor to low $q^2$ is obtained using the following expression [12]:

$$f_+(q^2) = F(q^2_{\text{max}}) \left( 1 + \frac{1}{6N} \xi^2 (q^2_{\text{max}} - q^2) \right)^{-N}$$

(1.46)

where $\xi$ is the charge radius of the final state meson, and $N = 2$ for decays to pseudo-scalar mesons and $N = 3$ for decays to vector mesons. The theoretical uncertainties for these calculations are very hard to quantify.

This model has been used as the default for the BABAR simulation of resonant hadronic states for all charmless semileptonic decays, and also for most decays involving charm mesons, except for $D^* \ell \nu$.

### 1.4.2 Light Cone Sum Rules

Light Cone Sum Rules (LCSR) provide a non-perturbative approach to form factor calculations that is complementary to lattice formulations, discussed in Sec.1.4.3, but also less rigorous. It combines the concept of QCD sum rules with twist expansions characteristic of hard exclusive processes in QCD [15]. The LCSR calculations provide estimates of various form factors for low to intermediate values of $q^2$ ($q^2 < 14 \text{ GeV}^2$), i.e., they cover that range of $q^2$ not covered by lattice results (see Sec.1.4.3).

The form factors have been parameterized in a universal form [18]

$$F(q^2) = \frac{F(0)}{1 - aq^2/m_B^2 + b(q^2/m_B^2)^2}$$

(1.47)

with three free parameters, $F(0)$, $a$, and $b$. This parameterization has been used to fit the prediction of the sum rules for $q^2 < 14 \text{ GeV}^2$ and it reproduces the calculated values with a 2% of accuracy.

For decays to scalar mesons, at larger $q^2$, a vector meson pole approximation can be used to extend the range of these calculations beyond a limit of $q^2_0$,

$$f_+(q^2) = \frac{c}{1 - q^2/m_B^2},$$

(1.48)

with $m_B = 5.32 \text{ GeV}^2$ and a boundary of $q^2_0 = 14 - 18 \text{ GeV}^2$. A proposed set of parameters for this approach for decays to scalar mesons is given in Tab. 1-2.

The LCSR calculations derive their overall normalization at low $q^2$, specifically [15, 18]

$$f_+^{\pi\ell\nu}(q^2 = 0) = 0.26 \pm 0.06$$

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Table 1-2. Parameterization of the QCD LCSR form factors calculations for decays to scalar mesons [15, 18].

<table>
<thead>
<tr>
<th>Form factor</th>
<th>$f_+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(0)$</td>
<td>0.26</td>
</tr>
<tr>
<td>$a$</td>
<td>2.03</td>
</tr>
<tr>
<td>$b$</td>
<td>1.29</td>
</tr>
<tr>
<td>$c$</td>
<td>0.44</td>
</tr>
<tr>
<td>$q_0^2$ [GeV$^2$]</td>
<td>15.7</td>
</tr>
</tbody>
</table>

where the error represents the uncertainties of the input parameters. There is an additional systematic uncertainty in the calculations that is estimated to be roughly ±0.05.

In 2004, Ball and Zwicky published new results [19, 20] of their LCSR calculations, which use the following parameterizations for decays to scalar mesons:

$$F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{fit}^2}$$  \hspace{1cm} (1.49)

with independent parameters $r_1$, $r_2$ and $m_{fit}$. Here $m_R$ is the mass of the lowest-lying resonance ($m_R = m_{BR(1-)} = 5.32$ GeV) and $m_{fit}$ is an effective pole mass. Tab.1-3 shows the suggested values for these parameters. The results are meant for $q^2 < 14$ GeV/c$^2$, beyond which LCSR is not valid. The theoretical uncertainty is quoted to be 10 – 13% at $q^2 = 0$.

1.4.3 Lattice QCD

Lattice QCD simulations potentially can provide heavy-to-light quark form factors from first principles. In practice, most calculations to date have been performed in the quenched approximation in which the effects of gluons and the quark-antiquark cloud surrounding the valence quarks are neglected. This introduces systematic uncertainties, estimated to be in the range of 10 to 15%. The FNAL [23] and HPQCD [24] collaborations have recently published two unquenched lattice calculations with significantly reduced uncertainties, which will be used in this analysis. The results of these calculations are displayed in Fig 1-4.
1.4 Theoretical model for the form factor calculations

Unquenched lattice QCD calculations published by the FNAL [23] and HPQCD [24] collaborations in 2004. The error bars include statistical and estimated systematic errors. The solid black line represents a combined fit of the Becirevic-Kaidalov [26] form-factor parameterization through all lattice points using the constraint $f_+(0) = f_0(0)$, and the dashed error band represents the uncertainty in $f_+$ obtained from the fit.

Lattice calculations are currently applicable for $q^2 > 16 \text{GeV}^2$, thus for small hadron momenta. Parameterizations of $f_+$ have to be used to extrapolate to low $q^2$.

Becirevic and Kaidalov (BK) [26] proposed a prescription for parameterizing $f_+$ in terms of both the nominal pole dominance of $B^*$ and a second effective pole that simulates the effect of higher resonance states. The BK parameterization is given by:

\begin{align}
    f_+(q^2) &= \frac{c_B (1 - \alpha)}{(1 - q^2/m_{B^*}^2)(1 - c_B q^2/m_{B^*}^2)} , \\
    f_0(q^2) &= \frac{c_B (1 - \alpha)}{1 - \beta^{-1} q^2/m_{B^*}^2} .
\end{align}

where $m_{B^*}$ is the $B^*$ mass and $c_B m_{B^*}^2$ is the residue of the form factor at $q^2 = m_{B^*}$.

Tab. 1-4 gives the values of the parameters $c_B$, $\alpha$, and $\beta$ from the original publication as well as the values obtained from fits to the recent lattice calculations. Fig. 1-5 compares the $q^2$ distribution for $\pi \ell \nu$ and $\rho \ell \nu$ decays for various form-factor calculations for the full lepton-energy range.
Table 1-4. Parameters of the BK parameterization as given in the original publication [26] or obtained from fits to the unquenched lattice calculations HPQCD’04 [24] and FNAL’04 [23] The errors are due to statistics and fitting.

<table>
<thead>
<tr>
<th></th>
<th>$c_B$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Becirevic &amp; Kaidalov</td>
<td>$0.8 \pm 0.3$</td>
<td>$0.54 \pm 0.17$</td>
<td>$1.4 \pm 0.9$</td>
</tr>
<tr>
<td>Fit to HPQCD’04</td>
<td>$0.42 \pm 0.03$</td>
<td>$0.41 \pm 0.07$</td>
<td>$1.18 \pm 0.05$</td>
</tr>
<tr>
<td>Fit to FNAL’04</td>
<td>$0.62 \pm 0.02$</td>
<td>$0.63 \pm 0.05$</td>
<td>$1.18 \pm 0.05$</td>
</tr>
</tbody>
</table>

Figure 1-5. $q^2$ distribution for $B \to \pi \ell \nu$ (left) for different form factor approximations, the MC default ISGW2, LCSR calculations, and unquenched LQCD calculations.

1.4.4 Theoretical uncertainties

The theoretical form factors uncertainties, that enter twice in the analysis on exclusive semileptonic decays, concern the form factors shape and normalization. The measurement of the $B \to X_u \ell \nu$ branching fractions is affected by the theoretical form factors uncertainties due to the shape, and the normalization is a source of uncertainty in the extraction of $|V_{ub}|$ from the measured $B \to X_c \ell \nu$ branching fractions.

Branching fraction measurements suffer from large theoretical uncertainties when the experimental efficiency varies significantly over the variables on which the form factor depend, typically the $q^2$. The various form factor calculations available in the literature show still a significant variation in $q^2$, as proved in Fig.1-5.

In the $B^0 \to \pi^- \ell^+ \nu$ decay there is a single form factor that dominates the dynamics for this mode in the limit of massless lepton, and therefore the $d\Gamma/dq^2$ distribution contains all of the dynamical information. The integration over a broad $q^2$ interval, combined with selection criteria which cause a significant efficiency variation in $q^2$, produces large differences in the weighting of low efficiency versus high efficiency $q^2$ regions. The overall efficiency is then strongly model dependent, leading to significant theoretical uncertainty.

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The effect of these uncertainties on the branching fraction can be reduced by minimizing the dependence of the experimental cuts on $q^2$ and by extracting the branching fractions in separate $q^2$ intervals.

In principle, the shape of the form factor can be measured experimentally on the data to reduce dependence on theoretical predictions, but high statistics are necessary. Instead, its normalization needs to be given by theoretical models which suffer from relatively large uncertainties. At the moment, the normalization of form factors is the largest source of uncertainty in the extraction of $|V_{ub}|$ from an exclusive $B \rightarrow X_u \ell \nu$ branching fraction.

The measurements presented in this thesis rely on these theoretical predictions for both the shape and the normalization.

1.5 Existing $|V_{ub}|$ measurements on charmless semileptonic decays

In this section a short review of the existing measurements of $|V_{ub}|$ is presented, focusing on the experimental aspects of each technique and giving an idea of the theoretical uncertainties associated to them.

1.5.1 Inclusive measurements

In an inclusive measurement of $|V_{ub}|$, one measures the rate of the charmless semileptonic decay $B \rightarrow X_u \ell \nu$ without reconstructing the hadronic system $X_u$.

The total charmless semileptonic rate is related to $|V_{ub}|$ through an expansion in powers of $1/m_b$ [27]:

$$\Gamma(b \rightarrow u \ell \nu) = \frac{G_F^2 m_b^7}{192\pi^3} |V_{ub}|^2 A_0 (1 - \frac{\mu^2 - \mu_G^2}{2m_b^2}) - 2\frac{\mu_G^2}{m_b^4} + O\left(\frac{1}{m_b^7}\right). \quad (1.52)$$

The theoretical description of inclusive $B \rightarrow X_u \ell \nu$ decays is based on the Heavy Quark Expansion, which predicts the total decay rate with uncertainties of about 5%.

Experimentally, the main challenge is to separate the signal $B \rightarrow X_u \ell \nu$ decays from the 50 times larger $B \rightarrow X_c \ell \nu$ background. This can be achieved by selecting regions of phase space in which this background is highly suppressed. In inclusive measurements the most common kinematic variables, discussed in literature, are the lepton energy ($E_\ell$), the hadronic invariant mass ($m_X$) and the leptonic invariant mass squared ($q^2 = (P_\ell + P_\nu)^2$). The spectra of these variables are affected by the distribution of the $b$ quark momentum inside the $B$ meson, which can be described by a structure or “shape function” (SF) [28, 29], as shown in Fig. 1-6, in addition to weak annihilation [30] and other non perturbative effects.

Since the SF reflects a universal property of $B$ mesons independent of the decay process, the SF parameters can be extracted from other $b \rightarrow$ light quark processes, as $B \rightarrow X_s \gamma$ or $B \rightarrow X_c \ell \nu$ decays, and then used into inclusive $B \rightarrow X_u \ell \nu$ decays.
Figure 1-6. The shape of the electron energy (left), hadronic invariant mass (center) and leptonic invariant mass (right) spectra. The dashed curves are the $b$ quark decay results to $O(\alpha_s)$, while the solid curves are obtained by convoluting the parton-level rate with the Fermi motion model with typical parameters. The vertical line marks a reasonable kinematic cut that can be used in order to discriminate $b \to u\ell\nu$ from $b \to c\ell\nu$ transitions. The singularities in $m_X^2$ spectrum reflect the unphysical nature of the parton level distributions. The differences vanish once the Fermi motion is implemented and the parton level variables are replaced with observable quantities.

Also the fraction $f_\alpha$ of the $B \to X_\alpha \ell\nu$ events that pass the experimental cuts depend strongly on the SF parameters, making them the largest source of uncertainty on $|V_{ub}|$. In order to estimate the impact of the SF on the charmless semileptonic rate in the reduced phase space region, several SF models have been proposed. These models are used in the theoretical calculation of $f_\alpha$, that heavily depends on the assumption for the differential decay width.

$|V_{ub}|$ is determined from the measurement of the charmless semileptonic partial branching fraction, $\Delta B(B \to X_\alpha \ell\nu)$, the $B$ meson lifetime, $\tau_B$, and the rate $\zeta(\Delta \Phi)$, which is predicted by theory (BLNP[31], DGE[32]) and depends on the phase space region, $\Delta \Phi$, defined by kinematic cuts:

$$|V_{ub}| = \sqrt{\frac{\Delta B(B \to X_\alpha \ell\nu)}{\tau_B \cdot \zeta(\Delta \Phi)}}. \quad (1.53)$$

The $B$ -factories have studied several kinematic variables to measure $|V_{ub}|$. The endpoint of the lepton energy spectrum for charmless decays is well above the one for charm decays ($E_\ell > 2.3 \text{ GeV}$). The good knowledge of the charm background allows to push this cut below the charm threshold, thereby increasing the acceptance and decreasing theory uncertainty. CLEO [33], BAbAR [34] and Belle [35] measured $|V_{ub}|$ using the lepton spectrum in the momentum region $2.3 - 2.6 \text{ GeV}/c$, $2.0 - 2.6 \text{ GeV}/c$ and $1.9 - 2.6 \text{ GeV}/c$ respectively.

Reconstructing unambiguously other variables involving either the neutrino or the $X$ system is experimentally challenging and requires more knowledge of the whole event. This can be achieved by reconstructing one $B$ in a pure hadronic mode and studying the recoiling $B$, whose momentum and flavor are then known. This technique provides signal over background ratio value of $\sim 1$ or higher, at the expense of a very small signal efficiency ($O(10^{-3})$).
Belle performed the measurement of $|V_{ub}|$ by using three different combinations of the common kinematic variables [36]. The $|V_{ub}|$ result obtained by BABAR, using the combined information of the $M_X - q^2$ distribution [37] agrees within errors with the Belle measurement.

All the inclusive $|V_{ub}|$ measurements are summarized in Fig. 1-7. The latest average from HFAG [38] using BLNP[31] gives $|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \cdot 10^{-3}$. Using the alternative approach by DGE[32] gives $|V_{ub}| = (4.46 \pm 0.20 \pm 0.20) \cdot 10^{-3}$. Both results agree very well and the total uncertainty on $|V_{ub}|$ is 7.3% using BLNP approach and 6.3% using DGE approach.

Recently BABAR measured inclusive $|V_{ub}|$ using the prescription of Leibovich, Low, and Rothstein (LLR) [39] that extracts $|V_{ub}|$ with reduced model dependence by combining data of the hadronic mass spectrum from $B \to X_u \ell \nu$ decays with that of the photon energy spectrum from $B \to X_s \gamma$ decays. The LLR approach reduces the dependence on the shape function because no model for the shape function is assumed, in contrast to the BLNP and DGE formulations used above. A comparison between the $|V_{ub}|$ measurement as a function of the different theoretical model is provided in Fig. 1-8.

### 1.5.2 Exclusive measurements

The theoretical framework of the exclusive semileptonic decays have been described in details in the previous sections.

The experimental approaches used in the exclusive measurements can be divided into classes: untagged and tagged techniques. The untagged and the semileptonic tag techniques and the existing measurements...
obtained using these experimental approaches are discussed in the following. The hadronic recoil technique will be described in detail later because it has been utilized in the analysis reported in this thesis.

The untagged technique, also called neutrino reconstruction technique, relies on exploiting the hermicity of the detector to infer the neutrino momentum from the missing momentum of the whole event. The method is denoted untagged since the $B$ meson recoiling against the signal candidate $B$ is not explicitly reconstructed. The major advantage of this method over the tagged techniques is a relatively high efficiency. The major disadvantage is that the resolution of the neutrino four-momentum is relatively poor, and this result in a lower purity and signal to background ratio.

In the tagged technique, one of the two $B$ mesons from the $\Upsilon (4S)$ decay is reconstructed in either a hadronic or semileptonic decay mode and the recoiling $B$ is studied to search for signature of the exclusive charmless semileptonic decay. This technique allows to constrain the kinematics, reduce the combinatorics and determine the charge of the signal $B$. Moreover, there is an improvement of the signal purity with respect to the untagged technique in which only the signal $B$ meson is reconstructed.

CLEO has a published result on $B \rightarrow \pi \ell \nu$ and $B \rightarrow \rho \ell \nu$ decays using an untagged analysis [40]. A similar analysis has been published by BABAR [41] and in progress in Belle. Recently, BABAR presented also a new measurement on $B \rightarrow \pi \ell \nu$ decays based on a novel technique denoted as loose neutrino reconstruction [42].

The traditional neutrino reconstruction has provided sufficiently large signal yields to allow some $q^2$ dependence information with CLEO’s dataset [40], and more with BABAR’s runs 1-2 dataset [41]. Anyway, the requirement that the signal neutrino is well reconstructed by vetoing events with additional missing particles causes important signal detection inefficiencies, that are avoided in the new technique developed by BABAR.

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Figure 1-9. Measured differential decay rate in 12 bins of $q^2$. The smaller uncertainty bars are statistical only while the larger uncertainty bars include statistical and systematic uncertainties. The BK [26] parameterization (solid black curve) reproduces the data quite well ($\chi^2 = 8.8$ for 11 degrees of freedom). The data are also compared to LCSR calculations [20] (dotted line), unquenched LQCD calculations [24] (black dashed line), [23] (grey dashed line) and the ISGW2 quark model [17] (dash-dot line).

The loose neutrino reconstruction is largely inspired by the traditional neutrino reconstruction: both methods use the missing momentum in the events as an approximation to the signal neutrino. The essential difference between the two approaches is that, in terms of four-momenta, the reconstructed neutrino is used to compute $q^2 = (P_L + P_\nu)^2$ in the traditional approach, while in the loose neutrino reconstruction $q^2 = (P_B - P_\pi)^2$ is determined. Although the relation $q^2 = (P_B - P_\pi)^2$ is strictly true and Lorentz invariant, it cannot be directly used because the value of $P_B$ is not known outside its rest frame. Only the value of $P_\pi$, measured in the laboratory frame and that of the $\Upsilon(4S)$ four-momentum are known. Nevertheless, since the $B$ momentum is small in the $\Upsilon(4S)$ frame, a common approximation is to boost the $\pi$ to the $\Upsilon(4S)$ frame and use the relation $q^2 = (P_B - P_\pi)^2$ in that frame, where the $B$ meson is assumed to be at rest.

The difference on the $q^2$ definition may appear to be small, but results in an increased signal efficiency. When using $q^2 = (P_L + P_\nu)^2$, the resolution of $q^2$ is completely dominated by the reconstructed neutrino: the more precise the $q^2$ resolution, the more stringent the neutrino quality cuts. $B$ mesons are produced in pairs at $B$ factories. It turns out that the neutrino quality cuts depend as much on the signal $B$ decay as on the other $B$ decay.

As a consequence, many neutrino quality cuts are not good at discriminating signal from background. The traditional neutrino reconstruction can improve the $q^2 = (P_L + P_\nu)^2$ resolution but not the signal/background ($S/B$) ratio, and therefore can lead to substantial signal yield loss. Tight neutrino quality cuts have also resulted in relatively large systematic uncertainties in the past because of their sensitivity to many aspects of the full event simulation. In an analysis with a large number of $q^2$ bins, this effect becomes important.

CKM Matrix and Semileptonic $B$ Decays
In the loose neutrino reconstruction technique, the reconstructed neutrino does not have to match the real signal neutrino with a good resolution because of the use of the relation $q^2 = (P_B - P_\nu)^2$.

Practically, the loose neutrino reconstruction is a fully untagged technique giving the largest signal yields. It depends on reconstructing the value of $q^2$ using the relation $q^2 = (P_B - P_\nu)^2$, and uses the neutrino information only to the extent that it can reject more background than signal. Using this new technique $\text{BABAR}$ performed the measurements of the partial branching fractions of $B^0 \rightarrow \pi^- \ell^+\nu$ decay mode in 12 bins of $q^2$. Experimental data has been used also to measure the form factor shape. Fig. 1-9 shows the shape dependence of the partial branching fraction for $B^0 \rightarrow \pi^- \ell^+\nu$ in 12 $q^2$ bins and a comparison of the measured form-factor shape with theoretical predictions. The data agree well with the recent LCSR [20] and unquenched LQCD [24, 23] results, but disfavor the widely used ISGW2 quark model [17].

Semileptonic tag method involves the partial reconstruction of a semileptonic $B$ meson decay to charm recoiling against the signal $B \rightarrow X_u \ell\nu$ candidate. Several $D$ and $D^*$ decay modes are used for the tagging. Since the final state contains a neutrino from both signal and tagging $B$, kinematic constraints must be employed to separate signal events from background. Backgrounds are lower than for the untagged method, but so is the efficiency. Both $\text{BABAR}$ [43] and Belle [44] published results on exclusive semileptonic decays using the semileptonic tag technique.

The hadronic tag, in which the $B$ meson recoiling against the signal $B$ candidate is fully reconstructed in a selected set of hadronic $B$ decay modes, assures a purity better than the other techniques. The relative small background allows for loose selection criteria and the theoretical uncertainties are highly reduced.

The results on the exclusive charmless semileptonic decays using the neutrino reconstruction and the semileptonic tag will be showed and discussed in Sec. 7.5, together with the measurements derived from the analysis with the hadronic tag, that is the topic of this thesis.
The BABAR Experiment

BABAR is a high energy physics experiment installed at the Stanford Linear Accelerator Center (SLAC), California. It was designed and built by a large international team of scientists and engineers in the 90s, with a comprehensive physics program consisting in the systematic measurement of CP violation in the $B$ meson system, precision measurements of decays of bottom and charm mesons and of the $\tau$ lepton, and search for rare processes. The experiment consists of a detector (BABAR [45]) built around the interaction region of a high luminosity $e^+e^-$ asymmetric collider (PEP-II [46]). In this chapter the main features of the final designs and the performances of PEP-II and the BABAR detector are described.

2.1 The PEP-II $B$ -Factory

The PEP-II $B$ -Factory is an asymmetric-energy $e^+e^-$ collider designed to operate at a center of mass energy of 10.58 GeV, corresponding to the mass of the $\Upsilon(4S) = b\bar{b}$ vector meson resonance.

The effective cross section\(^1\) for the production of the $\Upsilon(4S)$ at $\sqrt{s} = 10.58 \text{ GeV}$ is about 1.1 nb, and the $\Upsilon(4S)$ decays almost exclusively into $B^0\bar{B}^0$ or $B^+B^-$ pairs. The design peak luminosity was foreseen to be $\mathcal{L} = 3 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$, but during year 2004, thanks to higher beam currents, improved beam orbits and focusing, PEP-II has achieved a stable $\mathcal{L} = 9 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$, thus producing $B$ meson pairs at a rate of about 10 Hz, which translates to about 100 million $B\bar{B}$ pairs in one year of continuous running, and providing an ideal laboratory for the study of $B$ mesons.

The cross sections of the main physics processes in PEP-II are listed in Tab. 2-1 [47]. At the peak of the $\Upsilon(4S)$ there is a non-negligible amount of continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) and $e^+e^- \rightarrow \ell\ell$ ($\ell = e, \mu, \tau$) events. To study the background events due to these processes, part of the data is collected at a center of mass energy 40 MeV below the $\Upsilon(4S)$ peak, where $B\bar{B}$ production is not allowed. This data sample corresponds to about 1/10 of the sample taken at the $\Upsilon(4S)$ peak.

2.1.1 PEP-II layout

In PEP-II, the electron beam of 9.0 GeV collides head-on with the positron beam of 3.1 GeV resulting in a boost to the $\Upsilon(4S)$ resonance of $\beta\gamma \approx 0.56$ in the laboratory frame.

\(^1\)This effective cross section is lower (about one third) than the peak cross section (3.6 nb) due to the energy spread (3-6 MeV) of the beams and to initial state radiation.
Table 2-1. Cross sections of the main physics processes at the \( T(4S) \). The cross section for \( e^+e^- \) is referred to the volume of the BABAR electromagnetic calorimeter, which is used to trigger these events.

The asymmetry of the machine is motivated by the need of separating the decay vertices of the two \( B \) mesons, a feature that is crucial for the \( CP \) asymmetry determination. The boost makes it possible to reconstruct the decay vertices of the two \( B \) mesons and to determine their relative decay times, since the average separation between the two \( B \) vertices is \( \beta \gamma c T \approx 250 \mu m \). One can therefore measure the time dependent decay rates and \( CP \) asymmetries.

The parameters of PEP-II rings are summarized in Tab. 2-2.

Table 2-2. PEP-II beam parameters; both design values and values achieved in colliding beam operation during year 2005 are given. HER and LER refer to the high energy \( e^- \) and low energy \( e^+ \) ring, respectively. \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \) refer to the horizontal, vertical, and longitudinal c.m.s. size of the luminous region. The peak luminosity is proportional to \( E \eta_0 / \beta_y^* \).

Electrons and positrons are accelerated from the 3 km long SLAC linac and accumulated into two 2.2 km long storage rings, called HER (high-energy ring) and LER (low-energy ring) respectively. A fraction of electrons instead of being delivered to the HER is further accelerated to an energy of 30 GeV and sent to a target where positrons are produced. In proximity of the interaction region the beams are focused by a series of offset quadrupoles (Q1, Q2, Q4, Q5 in Fig. 2-3) and bent by means of a pair of samarium-
cobalt dipole magnets (B1), which allow the bunches to collide head-on. The tapered B1 dipoles, located at ± 21 cm on each side of the interaction point (IP), and the Q1 quadrupoles operate inside the field of the \textit{BABAR} superconducting solenoid, while Q2, Q4, and Q5, are located outside or in the fringe field of the solenoid.

The interaction region is enclosed in a water-cooled beam pipe consisting of two thin layers of beryllium (0.83 mm and 0.53 mm) with a 1.48-mm water channel in between. To attenuate synchrotron radiation, the inner surface of the pipe is gold-plated (approximately 4 \(\mu\)m). The total thickness of the central beam pipe section at normal incidence corresponds to 1.06 \% of a radiation length. The beam pipe, the permanent magnets and the Silicon Vertex Tracker (SVT) are assembled and aligned and then enclosed in a 4.4-m long support tube. This rigid structure is inserted into the \textit{BABAR} detector, spanning the IP.

### 2.1.2 PEP-II performances

Collisions in PEP-II started at the end of 1999, and since then \textit{BABAR} has recorded 21 million \(\Upsilon(4S)\) decays in RUN1 (Oct 1999 - Oct 2000), 66 million in RUN2 (Feb 2001 - Jun 2002), 34 million in RUN3 (Dec 2002 - Jun 2003) and 110 million in RUN4 (Sep 2003 - Jul 2004) and 199 million in RUN5 (Jan 2005 - June 2006) for a total of 430 million \(B\bar{B}\) pairs. The corresponding integrated luminosity is about 391 \(fb^{-1}\), while the luminosity integrated off-peak is about 37 \(fb^{-1}\). The actual \textit{BABAR} recorded luminosity is shown in Fig. 2-1.

![Graph showing PEP-II delivered and recorded integrated luminosity in RUN1 to RUN5.](image-url)

\textbf{Figure 2-1.} PEP-II delivered and \textit{BABAR}-recorded integrated luminosity in RUN1 to RUN5 (from October 1999 to August 2006).
As shown in Tab. 2-2 [46, 48], PEP-II has already surpassed its design performances, both in terms of the instantaneous luminosity and the daily integrated luminosity, achieving recently the peak value of $1 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}$ with a daily integrated luminosity of 700 $\text{pb}^{-1}$. Future upgrades that are currently being studied are expected to push the peak luminosity up to about $2.2 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}$ and will eventually allow the experiment to collect about 1 billion $\ell \bar{\ell}$ pairs by 2008. With this huge dataset the desired sensitivity for many exciting measurements of $CP$ -violating and rare $B$ decays will be reached. The progresses in the instantaneous luminosity are mainly due to the increase of the beam currents and improved focusing and beam orbits. A significant improvement to the integrated luminosity has been achieved between December 2003 and March 2004 with the implementation of a novel mode of operation of PEP-II, called “trickle injection”, which increases the production of $\ell \bar{\ell}$ pairs by up to 50 percent (Fig. 2-2). Until the end of 2003, PEP-II typically operated in a series of 40 minute fills during which the colliding beams coasted: at the end of each fill, it took about three to five minutes to replenish the beams for the next fill, and during this period the $\text{BABAR}$ data acquisition system had to be turned off for safety and dead-time reasons. With the new technique, the $\text{BABAR}$ detector can keep taking data virtually uninterrupted while the linac continuously injects electron and positron bunches (at a rate up to 10 Hz) into the two PEP-II storage rings to replace those that are lost in collisions in the $\text{BABAR}$ interaction region. After more than a year of testing, trickle injection was introduced first in the low energy ring in December 2003, bringing the $B$ Factory a 30% increase in output. In March 2004 also trickle injection for the high energy ring has been implemented, thus providing an additional 15% increase. The advantages of this novel mode of operation go beyond just the increase in luminosity: continuous injection makes the storage of particles more stable, so that PEP-II rings are easier to operate and beam losses are far less frequent than with the previous operational mode. This result is very important since, after a loss of the stored beams, it takes approximately 15 minutes to refill the two beams.
2.1.3 Machine Background

Beam-generated background causes high single counting rates, data acquisition dead times, high currents and radiation damages both of the detector components and of the electronics. This results in a lower data quality and may limit the lifetime of the apparatus. For this reason the background generated by PEP-II has been studied in detail and the interaction region has been carefully designed. Furthermore, background rates are continuously monitored during data acquisition to prevent critical operating conditions of the detector.

The primary sources of machine generated background [49] are:

- *synchrotron radiation* in the proximity of the interaction region. A strong source of background (many KW of power) is due to the beam deflections in the interaction region. This component is limited by channeling the radiation out of the BABAR acceptance with a proper design of the interaction region and the beam orbits, and placing absorbing masks before the detector components.

- *interaction* between beam particles and residual gas in either ring can have two different origins: beam-gas bremsstrahlung and Coulomb scattering. Both these two types of interaction causes an escape of the beam particle from their orbit. This background represents the primary source of radiation damage for the inner vertex detector and the principal background for the other detector components.

- *electromagnetic showers generated by beam-beam collisions*. These showers are due to energy degraded $e^+$ and $e^-$ produced by radiative Bhabha scattering and hitting the beam pipe within a few meters of the IP. This background is proportional to the luminosity of the machine and whereas now is under control, it is expected to increase in case of higher operation values.

2.2 Overview of the BABAR detector

The design of the BABAR detector is optimized for $CP$ violation studies, but it also well suited for searches of rare $B$ decays. To achieve the goal of performing accurate measurements there are many requirements:

- a large and uniform acceptance, in particular down to small polar angles relative to the boost direction, to avoid particle losses;

- excellent detection efficiency for charged particles down to 60 MeV/c and for photons down to 25 MeV;

- high momentum resolution to separate small signals from background;

- excellent energy and angular resolution for the detection of photons from $\pi^0$ and radiative $B$ decays in the range from 25 MeV to 4 GeV;

- very good vertex resolution, both transverse and parallel to the beam;
The BABAR detector (Fig. 2-3), designed and fabricated by a collaboration of 600 physicists of 75 institutions from 9 countries, meets all these requirements, as will be shown in the next sections of this chapter.

An overview of the polar angle ($\theta$) coverage, the segmentation and performance of the BABAR detector systems is summarized in Tab. 2-3. The BABAR superconducting solenoid, which produces a 1.5 T axial magnetic field, contains a set of nested detectors, which are – going from inside to outside – a five layers Silicon Vertex Tracker (SVT), a central Drift Chamber (DCH) for charged particles detection and momentum measurement, a fused-silica Cherenkov radiation detector (DIRC) for particle identification, and a CsI(Tl) crystal electromagnetic calorimeter for detection of photons and electrons. The calorimeter has a barrel and an end-cap which extends it asymmetrically into the forward direction ($\phi$ beam direction), where many of the collision products emerge. All the detectors located inside the magnet have full acceptance in azimuth ($\phi$).

The flux return outside the cryostat is composed of 18 layers of steel, which increase in thickness outwards, and are instrumented (IFR) with 19 layers of planar resistive plate chambers (RPCs) or limited streamer tubes (LSTs) in the barrel and 18 in the end-caps. The IFR allows the separation of muons and charged hadrons, and also detect penetrating neutral hadrons. As indicated in Fig.2-3, the right-handed coordinate system is anchored to the main tracking system, the drift chamber, with the $z$-axis coinciding with its principal axis. This axis is offset relative to the beam axis by about 20 mrad in the horizontal plane.

**Table 2-3.** Overview of the coverage, segmentation, and performance of the BABAR detector systems. The notation (C), (F), and (B) refers to the central barrel, forward and backward components of the system, respectively. Performance numbers are quoted for 1 GeV/c particles, except where noted.

<table>
<thead>
<tr>
<th>System</th>
<th>Polar angle coverage</th>
<th>Channels</th>
<th>Layers</th>
<th>Segmentation</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVT</td>
<td>[20.1,150.2]°</td>
<td>150K</td>
<td>5</td>
<td>50-100μm, $\phi = \phi$</td>
<td>$\sigma_{dE} = 5\mu$m, $\sigma_{\phi} = 65\mu$m</td>
</tr>
<tr>
<td>DCH</td>
<td>[17.2,152.6]°</td>
<td>7.104</td>
<td>40</td>
<td>6.8mm drift distance</td>
<td>$\sigma_{\phi} = 1 \text{ m rad}$, $\sigma_{\phi}/\sigma_{\phi} = 0.001$, $\sigma_{dE}/dE = 7.5%$</td>
</tr>
<tr>
<td>DIRC</td>
<td>[25.5,141.4]°</td>
<td>10,752</td>
<td>1</td>
<td>350×130 m² ($\Delta\phi \times \Delta r$)</td>
<td>$\sigma_{dE} = 2.5 \text{ m rad}$ per track</td>
</tr>
<tr>
<td>EMC(C)</td>
<td>[27.1,140.8]°</td>
<td>2×5700</td>
<td>1</td>
<td>47×470 m² $\Delta r$</td>
<td>$\sigma_{dE}/E = 3.6%$, $\sigma_{\phi} = 3.9 \text{ m rad}$</td>
</tr>
<tr>
<td>EMC(F)</td>
<td>[15.8,27.1]°</td>
<td>2×800</td>
<td>1</td>
<td>5760 crystals</td>
<td>$\sigma_{\phi} = 3.9 \text{ m rad}$</td>
</tr>
<tr>
<td>IFR(C)</td>
<td>[147,123]°</td>
<td>22K+2K</td>
<td>18</td>
<td>20-38 mm</td>
<td>90% $\mu^{\pm}$ eff., 6.8% $\mu^{\pm}$ mis-id (loose selection, 1.5-3.0 GeV/$c$)</td>
</tr>
<tr>
<td>IFR(F)</td>
<td>[20,47]°</td>
<td>14.5K</td>
<td>18</td>
<td>28-38 mm</td>
<td>6-8% $\pi^\pm$ mis-id</td>
</tr>
<tr>
<td>IFR(B)</td>
<td>[123,154]°</td>
<td>14.5K</td>
<td>18</td>
<td>28-38 mm</td>
<td></td>
</tr>
</tbody>
</table>

- identification of electrons and muons over a range of momentum, primarily for the detection of semi-leptonic decays used to tag the $B$ flavor and for the study of semi-leptonic and rare decays;
- identification of hadrons over a wide range of momentum for $B$ flavor tagging as well as for the separation of pions from kaons in decay modes like $B^0 \to K^+\pi^-$ and $B^0 \to K^+\pi^-$ as well as in charm meson and $\tau$ decays;
- a highly efficient, selective trigger system with redundancy so as to avoid significant signal losses and systematic uncertainties.

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Figure 2-3.  BABAR detector front view (top) and side view (bottom).
The positive $y$-axis points upward and the positive $x$-axis points away from the center of the PEP-II storage rings.

Since the average momentum of charged particles produced in $B$ meson decay is below $1 \text{ GeV}/c$, the errors on the measured track parameters are dominated by multiple Coulomb scattering, rather than the intrinsic spatial resolution of the detectors. Similarly, the detection efficiency and energy resolution of low energy photons are severely impacted by material in front of the calorimeter. Thus, special care has been given to keep the material in the active volume of the detector to a minimum. Fig. 2-4 shows the distribution of material in the various detector systems in units of radiation lengths. Specifically, each curve indicates the material a particle traverses before it reaches the first active element of a specific detector system.

### 2.3 The Silicon Vertex Tracker

The Silicon Vertex Tracker (SVT) provides a precise measurement of the decay vertices and of the charged particle trajectories near the interaction region. The mean vertex resolution along the $z$-axis for a fully reconstructed $B$ decay must be better than $80 \mu m$ in order to avoid a significant impact on the time-dependent $CP$ asymmetry measurement precision; a $100 \mu m$ resolution in the $x - y$ transverse plane is necessary in reconstructing decays of bottom and charm mesons, as well as $\tau$ leptons.

The SVT also provides standalone tracking for particles with transverse momentum too low to reach the outer tracker, like soft pions from $D^*$ decays and many charged particles produced in multi-body $B$ meson decays. The choice of a vertex tracker made of five layers of double-sided silicon strip sensors allows a complete track reconstruction even in the absence of the drift chamber information.

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Finally, the SVT supplies particle identification (PID) information both for low and high momentum tracks. For low momentum tracks the SVT $dE/dx$ measurement is the only PID information available, for high momentum tracks the SVT provides the best measurement of the track angles, required to achieve the design resolution on the Cherenkov angle measured by the DIRC.

### 2.3.1 Detector layout

The Silicon Vertex Tracker is composed of five layers of 300 $\mu$m thick, double-sided micro-strip detectors [50]. The total active silicon area is 0.96 m$^2$ and the material traversed by particles at normal incidence is 4% $X_0$. The geometrical acceptance is 90% of the solid angle in the center of mass system.

The silicon detectors and the associated readout electronics are assembled into mechanical units called *modules*. The inner three layers are barrel-shaped and are composed by six modules each. They are placed next to the interaction region, at radii 3.3, 4.0 and 5.9 cm from the beam axis (Fig. 2-5) and provide an accurate measurement of the track impact parameters along $z$ and in the $x-y$ plane. The outer two layers, composed by 16 and 18 modules (Fig. 2-5), have a peculiar arch structure to reduce the incident angles of particles going in the forward and backward direction; their barrel parts are placed at radii between 12.7 and 14.6 cm from the beam axis. They permit an accurate polar angle measurement and, along with the inner three layers, enable stand-alone tracking for particles with low transverse momentum $p_T$. Full azimuthal coverage is obtained by partially overlapping adjacent modules, either by tilting them in $\phi$ by 5° (inner layers) or by staggering them (outer layers); this overlap is also advantageous for alignment. The polar angle coverage is $20.1^\circ < \theta_{ab} < 150.2^\circ$.

Each silicon detector consists of a high-resistivity $n^-$ bulk on which are implanted $p^+$ strips on one side and orthogonally-oriented $n^+$ strips on the other side. The strips are AC-coupled to the electronics via integrated decoupling capacitor. The detectors are operated in reverse mode at full depletion, with bias voltage $V_{\text{bias}}$ typically 10 V higher than the depletion voltage $V_{\text{dep}}$ (which lies in the range 25 V – 35 V). The strips are biased through polysilicon resistors (4-20 M$\Omega$) and the detector active area is surrounded by an implanted guard ring that collects the edge currents and shapes the electric field in the active region. The $n^+$ strips insulation is provided by surrounding each $n^+$ strip with a $p$ implant called *p-stop*, so as to achieve an inter-
strip resistance greater than 100 MΩ at the operating bias voltage. The strip readout pitch varies with the layer and the side of the sensors (z, φ) from a minimum of 50 μm to a maximum of 210 μm.

2.3.2 Detector performance

2.3.2.1 Hit efficiency and resolution

The SVT hit efficiency is determined by comparing the number of hits found by a half-module and assigned to a reconstructed track with the number of tracks that cross the half-module. Excluding 5 out of 208 defective readout sections, the combined hardware and software efficiency is measured to be about 97%.

Fig. 2.6(a) shows the measured SVT spatial hit resolution in z and τ − φ for the first layers, as a function of the track incident angle with respect to the silicon wafer plane. The resolution is determined from the distribution of the distance in the wafer plane between the hit and the track trajectory of high-momentum tracks. The track trajectory uncertainty contribution is subtracted to obtain the hit resolution, which varies between 15 and 50 microns.

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2.3 The Silicon Vertex Tracker

2.3.2.2 $B$ decay vertex resolution

Figure 2.6(b) [51] shows the estimated error in the measurement of the difference along the $z$-axis between the vertices of two neutral $B$ mesons, one of them being fully reconstructed and the other one only partially for flavor-tagging purposes. The RMS width of the distribution, equal to $190\mu m$, meets the design expectation. It is dominated by the reconstruction of the tagging $B$ vertex, the RMS vertex resolution for fully reconstructed $B$ mesons being only $70\mu m$.

2.3.2.3 Tracking efficiency and track parameter resolution

The SVT provides stand-alone tracking for low momentum particles that do not reach the drift chamber, such as soft pions originating in decays of $D^*$ meson produced in $B\bar{B}$ events. A comparison of the detected slow pion spectrum with the Monte Carlo prediction is presented in Fig. 2.7(a) [45]. Based on this very good agreement the detection efficiency is estimated to be 20% for particles with transverse momenta of 50 MeV/c, rapidly increasing to over 80% at 70 MeV/c.

Figure 2-7. SVT detector performances: Tracking efficiency and track parameter resolution (a) and $dE/dx$ resolution (b).

(a) Data-Monte Carlo comparison of the transverse momentum spectrum of soft pions in $D^{*+} \rightarrow D^0 \pi^+$ (top), and efficiency for slow pions detection estimated from simulated events (bottom).

(b) Energy loss per unit length ($dE/dx$) as measured in the SVT as a function of momentum. The vertical scale is arbitrary.
2.3.2.4 \(dE/dx\) resolution

Limited particle ID information for low momentum particles that do not reach the drift chamber and the Cherenkov detector is provided by the SVT through the measurement of the specific ionization loss, \(dE/dx\), as derived from the total charge deposited in each silicon layer. It is computed as a truncated mean from the lowest 60% of the individual \(dE/dx\) measurements for tracks with at least 4 associated SVT hits. The resulting SVT \(dE/dx\) distribution as a function of momentum is shown in Fig.2.7(b) [52]. The superimposed Bethe-Bloch curves for the individual particle species have been determined using various particle control samples. The resolution achieved to date is typically about 14% for minimum ionizing particles, and a 2\(\sigma\) separation between kaons and pions can be achieved up to momenta of 500 MeV/c.

2.4 The Drift Chamber

The Drift Chamber (DCH) is the main tracking device for charged particles with transverse momenta \(p_T\) above \(\approx 120\) MeV/c, providing the measurement of \(p_T\) from the curvature of the particle’s trajectory inside the 1.5 T solenoidal magnetic field. The DCH also allows the reconstruction of secondary vertices located outside the silicon detector volume, such as those from \(K^0_s \rightarrow \pi^+ \pi^-\) decays. For this purpose, the chamber is able to measure not only the transverse coordinate, but also the longitudinal \(z\) position of tracks with good \(\approx 1\) mm resolution. Good \(z\) resolution also aids in matching DCH and SVT tracks, and in projecting tracks to the DIRC and the calorimeter.

For low momentum particles the DCH provides particle identification by measurement of ionization loss \((dE/dx)\), thus allowing for \(K/\pi\) separation up to \(\approx 700\) MeV/c. This capability is complementary to that of the DIRC in the barrel region, while it is the only mean to discriminate between different particle types.
hypotheses in the extreme backward and forward directions which fall outside of the geometric acceptance of the DIRC.

Finally, the DCH provides real-time information to the charged particle trigger.

### 2.4.1 Detector layout

The final design adopted for the Drift Chamber, illustrated in Fig. 2-8, consists of a 280 cm-long cylinder located within the volume inside the DIRC and outside the PEP-II support tube [53]. The inner radius is 23.6 cm and the outer radius is 80.9 cm. To take into account PEP’s asymmetric boost, the center of the chamber is displaced in the forward direction with respect to the IP by 36.7 cm, thus increasing the acceptance for forward-going tracks. The active volume provides charged particle tracking over the polar angle range $17.2^\circ < \theta_{\text{lab}} < 152.6^\circ$.

The drift system consists of 7104 hexagonal cells, approximately 1.8 cm wide by 1.2 cm high, arranged in 40 concentric layers. Each hexagonal cell consists of one sense wire surrounded by six field-shaping wires. In such a configuration an approximate circular symmetry of the equipotential contours is reached over a large portion of the cell. The field wires are at ground potential while high positive voltage is applied to the sense wire.

The 40 layers are grouped by 4 into super-layers. Fig.2-9 shows the four innermost super-layers. A complete symmetry along the $z$-axis does not allow the track position reconstruction along that direction. For this reason two different wire types are used: the type wire A, parallel to the $z$-axis, provides position measurements in the $x-y$ plane, while longitudinal position information is obtained with wires placed at small angles with respect to the $z$-axis (U or V wire type) (see Fig.2-10). Sense and field wires have the same orientation in each super-layer and are alternating following the scheme AUVAUVAUVA.

The 40 layers provide up to 40 spatial and ionization loss measurements for charged particles with $p_T$ greater than 180 MeV/c. In order to reduce the impact of multiple scattering on $p_T$ resolution, material within the chamber volume has been minimized (0.2% $X_0$) using low-mass aluminum field-wires and a helium-based gas mixture. The main properties of the gas system are listed in Tab. 2-4. The inner wall has been kept thin (0.28% $X_0$) to improve the contribution of the high-precision measurement in the outer layer of the SVT to the $p_T$ resolution, and minimize backgrounds due to photon conversions in the chamber wall. Material in the outer wall has also been minimized (0.6% $X_0$) so as not to degrade the DIRC and the EMC performances.

### 2.4.2 Detector performance

#### 2.4.2.1 Tracking efficiency and resolution

The drift chamber reconstruction efficiency has been measured on data in selected samples of multi-track events by exploiting the fact that tracks can be reconstructed independently in the SVT and the DCH. The absolute drift chamber tracking efficiency is determined as the fraction of all tracks detected in the SVT
Figure 2-9. Schematic layout of drift cells for the four innermost super-layers. Lines have been added between field wires to aid in visualization of the cells boundaries. The numbers on its right side give the stereo angles (mrad) of sense wires in each layer. The 1 mm-thick beryllium inner cylinder is also shown inside the first layer.

which are also reconstructed by the DCH when they fall within its acceptance. Its dependency on the transverse momentum and polar angle is shown in Fig. 2-11 [45]. At the design voltage of 1960V the reconstruction efficiency of the drift chamber averages $98 \pm 1\%$ for tracks above $200$ MeV/$c$ and polar angle $\theta > 500$ mrad ($29^\circ$). At the typical operating voltage of 1930V it decreases by about 2%.

The $p_T$ resolution, directly related to the curvature ($\omega$) resolution, is measured as a function of $p_T$ in cosmic ray studies (see Fig. 2-12 [54]). The data are well represented by a linear function:

$$\frac{\sigma_{p_T}}{p_T} = (0.13 \pm 0.01)\% \cdot p_T + (0.45 \pm 0.03)\%, \quad (2.1)$$

where $p_T$ is measured in GeV/$c$. The first contribution, dominating at high $p_T$, comes from the curvature error due to finite spatial measurement resolution; the second contribution, dominating at low momenta, is due to multiple Coulomb scattering.

2.4.2.2 $dE/dx$ Resolution

The specific ionization loss $dE/dx$ for charged particles traversing the drift chamber is derived from the total charge deposited in each drift cell. It is computed as a truncated mean from the lowest 80% of the individual $dE/dx$ measurements; various corrections are applied to remove several sources of bias (such, for instance, changes in gas gain due to temperature and pressure variations) that would degrade the accuracy of the

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2.4 The Drift Chamber

Figure 2-10. Drift cell isochrones (contours of equal drift times of ions) in cells of layers 3 and 4 of an axial super-layer. The isochrones are spaced by 100 ns. They are circular near the sense wires, but becomes irregular near the field wires, and extend into the gap between super-layers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture He : C₄H₁₀</td>
<td>80:20</td>
</tr>
<tr>
<td>Radiation Length</td>
<td>807 m</td>
</tr>
<tr>
<td>Primary Ions (m.i.p.)</td>
<td>21.2/cm</td>
</tr>
<tr>
<td>Drift Velocity</td>
<td>22 μm/ns</td>
</tr>
<tr>
<td>Avalanche gain</td>
<td>5 × 10⁴</td>
</tr>
<tr>
<td>Lorentz Angle</td>
<td>32°</td>
</tr>
<tr>
<td>dE/dx Resolution</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

Table 2-4. Properties of helium-isobutane gas mixture at atmospheric pressure and 20°C (in BaBar the gas is operated at a small over pressure of 4 mbar). The drift velocity is given for operation without magnetic field, while the Lorentz angle is stated for a 1.5 T magnetic field. The anode-cathode operating potential difference is 1960 V.

primary ionization measurement. The left plot of Fig. 2-13 shows the distribution of the reconstructed and corrected dE/dx from the drift chamber as a function of track momenta. The superimposed Bethe-Bloch curves for the individual particle species have been determined using various particle control samples. The resolution achieved to date is typically about 7.5% (as shown in the right plot of Fig. 2-13 for e± from Bhabha scattering), limited by the number of samplings and Landau fluctuations. A 3σ separation between kaons and pions can be achieved up to momenta of about 700 MeV/c [54].

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Figure 2-11. Track reconstruction efficiency in the drift chamber at operating voltages of 1900 V and 1960 V, as a function of transverse momentum (a) and polar angle (b).

Figure 2-12. $p_T$ resolution determined from cosmic ray muons.
2.5 The Cherenkov light detector

The particle identification (PID) at low momenta exploits primarily the $dE/dx$ measurements in the DCH and SVT. However, above the threshold of 700 MeV/$c$, the $dE/dx$ information does not allow to separate pions and kaons. BABAR has therefore a dedicated PID subdetector. The Detector of Internally Reflected Cherenkov radiation (DIRC) is employed primarily for the separation of pions and kaons from about 500 MeV/$c$ to the kinematic limit of 4 GeV/$c$. It was designed to be able to provide $K/\pi$ separation of $\approx 3\sigma$ or greater, for all tracks from $B$ meson decays from the pion Cherenkov threshold up to 4.2 GeV/$c$.

2.5.1 Detector layout

The DIRC is a novel type of ring-imaging Cherenkov detector, based on the principle that the magnitudes of angles are maintained upon reflection from a flat surface [55]. Fig. 2-14 shows a schematic of the DIRC geometry, while Fig. 2-15 illustrates the principles of light production, transport, and imaging.

The radiator material of the DIRC is synthetic fused silica (refraction index $n = 1.473$) in the form of 144 long, thin bars with regular rectangular cross section. The bars, which are 17-mm-thick, 35-mm-wide and 4.9-m-long, are arranged in a 12-sided polygonal barrel, each side being composed of 12 adjacent bars. The solid angle subtended by the radiator bars corresponds to 94% of the azimuth and 83% of the cosine of the polar angle in the center-of-mass system. The total thickness of the DIRC material (bars and support structure) at normal incidence ($\theta = 90^\circ$) is only 8 cm, corresponding to $17\% X_0$. Such a thin Cherenkov detector allows to have, at the same time, a large inner tracking volume, which is needed to achieve the desired momentum resolution, and a compact outer electromagnetic calorimeter, with improved angular resolution and limited costs.
The bars serve both as radiators and as light pipes for the portion of the light trapped in the radiator by total internal reflection (the internal reflection coefficient of the bar surfaces is greater than 0.9992 per bounce). A charged particle with velocity $v > c/n$, traversing the fused silica bar, generates a cone of Cherenkov photons of half-angle $\theta_c$ with respect to the particle direction, where $\cos \theta_c = 1/\beta n$, $\beta = v/c$. For particles with $\beta \approx 1$, some photons will always lie within the total internal reflection limit, and will be transported to either one or both ends of the bar, depending on the particle incident angle. To avoid having to instrument both bar ends with photon detectors, a mirror is placed at the forward end, perpendicular to the bar axis, to reflect incident photons to the backward (instrumented) bar end.

Once photons arrive at the instrumented end, most of them emerge into an expansion region filled with 6000 litres of purified water ($n = 1.346$), called the stand-off box. A fused silica wedge at the exit of the bar reflects photons at large angles and thereby reduces the size of the required detection surface. The photons are detected by an array of densely packed photo-multiplier tubes (PMTs), each surrounded by reflecting “light catcher” cones to capture light which would otherwise miss the PMT active area. The PMTs, arranged in 12 sectors of 896 photo-tubes each, have a diameter of 29 mm and are placed at a distance of about 1.2 m from the bar end. The expected Cherenkov light pattern at this surface is essentially a conic section, whose cone opening-angle is the Cherenkov production angle modified by refraction at the exit from the fused silica window.

The time taken for the photon to travel from the point of origin to the PMT is also related to the photon propagation angle $(\alpha_x, \alpha_y, \alpha_z)$ with respect to the bar axis. As the track position and angles are known from the tracking system, these three $\alpha$ angles can be used to (over-)determine the Cherenkov angle $\theta_c$. This

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over-constraint on the angles is particularly useful in suppressing hits from beam-generated background and from other tracks in the same event, and also in resolving some ambiguities in the association between the PMT hits and the track (for instance, the forward-backward ambiguity between photons that have or haven’t been reflected by the mirror at the forward end of the bars). The relevant observable to distinguish between signal and background photons is the difference between the measured and expected photon time, $\delta t_{\gamma}$. It is calculated for each photon using the track-time of the PMT and the photon propagation time within the bar and the water filled standoff box. The resolution on this quantity, as measured in dimuon events (Fig. 2-17(b) [45]), is 1.7 ns, close to the intrinsic 1.5 ns transit time spread of the photoelectrons in the PMTs. Applying the time information substantially improves the correct matching of photons with tracks and reduces the number of accelerator induced background hits by approximately a factor 40, as can be seen in Fig. 2-16 [56].

2.5.2 Detector performance

In the absence of correlated systematic errors the resolution $\sigma_{\theta_{c,\text{track}}}$ on the track Cherenkov angle scale as

$$\sigma_{\theta_{c,\text{track}}} = \sigma_{\theta_{c,\gamma}}/\sqrt{N_{\gamma}},$$  \hspace{1cm} (2.2)

where $\sigma_{\theta_{c,\gamma}}$ is the single photon Cherenkov angle resolution and $N_{\gamma}$ is the number of photons detected.

The single photon Cherenkov angle resolution has been measured in dimuon events to be 10.2 mrad (Fig. 2-17(a) [45]). The main contributions to it come from the geometry of the detector (the size of the bars, the diameter of the PMTs and the distance between the bars and the PMTs give a 7 mrad contribution) and from the spread of the photon production angle, dominated by a 5.4 mrad chromatic term. Fig. 2-18 shows the number of photons detected as a function of the polar angle. It increases from a minimum of about 20 at the center of the barrel ($\theta \approx 90^\circ$) to well over 50 in the forward and backward directions, corresponding to the fact that the path-length in the radiator is longer for tracks emitted at large dip angles (therefore the

![Figure 2-15. Schematics of the DIRC fused silica radiator bar and imaging region.](image-url)
number of Cherenkov photons produced in the bars is greater) and the fraction of photons trapped by total internal reflection rises. This feature is very useful in the BABAR environment, where - due to the boost of the center-of-mass - particles are emitted preferentially in the forward direction. The bump at $\cos \theta = 0$ is a result of the fact that for tracks at small angles internal reflection of the Cherenkov photons occurs in both the forward and backward direction. The small decrease of the number of photons from the backward direction to the forward one is a consequence of the photon absorption along the bar before reaching the stand-off box in the backward end. The combination of the single photon Cherenkov angle resolution, the distribution of the number of detected photons versus polar angle and the polar angle distribution of charged tracks yields a typical track Cherenkov angle resolution which is about $2.5 \text{ mrad}$ for muons in dimuon events. A similar average resolution is found for charged kaons and pions in a sample of 430000 $D^{*+} \rightarrow D^0 \pi^+ (D^0 \rightarrow K^- \pi^+)$ decays reconstructed in data, where $K^\mp/\pi^\pm$ tracks are identified through the charge correlation with the $\pi^\pm$ from the $D^{*\pm}$ decay. From the measured single track resolution versus momentum and the difference between the expected Cherenkov angles of charged pions ($\theta_C^\pi$) and kaons ($\theta_C^K$), the pion-kaon separation power of the DIRC, $|\theta_C^K - \theta_C^\pi|/\sigma_{\text{theta}_C}$, can be inferred. As shown in Fig. 2-19, the separation between kaons and pions at 3 GeV/$c$ is about 4.3 $\sigma$.

### 2.6 The Electromagnetic Calorimeter

The BABAR electromagnetic calorimeter (EMC) is designed to detect and measure electromagnetic showers with high efficiency and very good energy and angular resolution over an energy range between 20 MeV
2.6 The Electromagnetic Calorimeter

Figure 2-17. Difference between (a) the measured and the expected Cherenkov angle for single photons and (b) the measured and expected photon arrival time, as measured in muons produced in dimuon events.

Figure 2-18. Number of detected photoelectrons versus track polar angle for reconstructed di-muon events in data and simulation.

(low-energy photons from $\pi^0$ mesons originating in $B$ decays) and 9 GeV (electrons from Bhabha scattering). It is also the primary sub-detector providing electron-hadron separation.

Energy deposit clusters in the EMC with lateral shape consistent with the expected pattern from an electromagnetic shower are identified as photons when they are not associated to any charged tracks extrapolated from the SVT and the drift chamber, and as electrons if they are matched to a charged track and the ratio between the energy $E$ measured in the EMC and the momentum $p$ measured by the tracking system is $E/p \approx 1$. 

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Figure 2-19. (a) The measured Cherenkov angle for pions (upper band) and kaons (lower band) from $D^* \to D^0 \pi$, $D^0 \to K\pi$ decays reconstructed in data. The curves show the expected angle $\theta_C$ as a function of laboratory momentum, for the $K$ and $\pi$ mass hypothesis. (b) The average difference between the expected value of $\theta_C$ for kaons and pions, divided by the uncertainty, as a function of momentum.

The efficient reconstruction of extremely rare decays of $B$ mesons containing $\pi^0$ s (e.g. $B^0 \to \pi^0 \pi^0$) poses the most stringent design requirements on energy resolution of order 1% while excellent photon efficiency at low energy ($\sim 20$ MeV) is required for efficient reconstruction of $B$ meson decays containing multiple $\pi^0$ and $\eta$. Similar precision is required for efficient separation of electrons and hadrons with purities required at the 0.1% level for momentum as low as 500 MeV/$c$. The $\pi^0$ mass resolution is dominated by the energy resolution at low energies (below 2 GeV) and by the angular resolution at high energies (above 2 GeV). The angular resolution is required to be a few milliradians in order to maintain good $m_\pi^0$ resolution ($\sigma_{m_\pi^0} \approx 6.5$ MeV) at all energies. The need for high efficiency requires hermetic coverage of the acceptance region while excellent resolution is achieved by minimizing the material in front of and between the active detector elements.

2.6.1 Detector layout

The BABAR electromagnetic calorimeter (Fig. 2-20) is a total-absorption calorimeter composed of 6580 CsI crystals doped with thallium iodide at about 1000 ppm [57]. The main properties of CsI(Tl) are summarized in Tab. 2-5: the high light yield and small Molière radius give the excellent energy and angular resolution required, while the short radiation length guarantees complete shower containment at BABAR energies with a relatively compact design. Furthermore, the high light yield and peak of the emission spectrum permit an efficient use of a silicon photodiode readout.

Each crystal is a truncated trapezoidal pyramid, with thickness between 29.6 cm (16 $X_0$) and 32.4 cm (17.5 $X_0$) and typical front face area $5 \times 5 cm^2$. The crystals are arranged quasi-projectively in a barrel structure of 48 $\theta$ rows by 120 crystals in azimuth ($\phi$), with an inner radius of 90 cm. The forward end is closed by a separable end-cap capable of holding nine additional rows. This geometry provides full azimuthal
2.6 The Electromagnetic Calorimeter

2.6.1 Coverage, while the polar angle coverage is $15.8^\circ < \theta_{lab} < 140.8^\circ$. To minimize the material in front of the calorimeter, the support structure of the crystals (which is made in carbon fiber) and the front-end electronics are located at the outer radius of the EMC. To recover the small fraction of light that is not internally reflected by the crystal surface, each crystal is wrapped with a diffuse reflective material (TYVEK). The scintillation light generated inside each crystal is detected by two independent $2 \times 1/\text{cm}^2$ silicon PIN photodiodes epoxied to its rear face.

<table>
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</thead>
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</tr>
<tr>
<td>Molière Radius</td>
<td>3.8 cm</td>
</tr>
<tr>
<td>Density</td>
<td>4.53 g/cm$^3$</td>
</tr>
<tr>
<td>Light Yield</td>
<td>50000 $\gamma$/MeV</td>
</tr>
<tr>
<td>Light Yield Temperature Coefficient</td>
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</tr>
<tr>
<td>Peak Emission $\lambda_{\text{max}}$</td>
<td>565 nm</td>
</tr>
<tr>
<td>Refractive Index ($\lambda_{\text{max}}$)</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Table 2-5. Properties of CsI(Tl).

2.6.2 Detector performance

2.6.2.1 Energy resolution

The limiting energy resolution of a homogeneous calorimeter is determined by fluctuations in the electromagnetic shower propagation and in the case of the BABAR crystal detector is empirically described as the

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The stochastic term $\sigma_1 E^{-\frac{1}{2}}$, which is dominant at low energies, arises primarily from fluctuations in photon statistics, but depends also on electronic noise in the readout chain and on the presence of beam-generated background. The constant term $\sigma_2$, dominant at higher energies, arises from several effects of which the main are fluctuations in shower containment due to leakage out the rear of the crystal or absorption in the material between and in front of the crystals, and uncertainties in the calibrations.

In BABAR the energy resolution as a function of energy is measured on data on selected control samples, including electrons and positrons from Bhabha scattering (energies between 3 and 9 GeV), photons from $\pi^0$ and $\eta$ decays (energies below 2 GeV) and from the decay $\chi_{c1} \rightarrow J/\psi \gamma$ ($E \approx 500$ MeV). At low energies the resolution is determined through weekly calibrations performed with a radioactive source ($^{16}O^6$) of 6.13 MeV photons. A fit to the resolution dependence on the energy with the empirical parameterization of Eq. (2.3), shown in Fig. 2-21(a) [57], yields:

$$\frac{\sigma_E}{E} = \frac{(2.32 \pm 0.30)\%}{\sqrt{E(\text{GeV})}} \oplus (1.85 \pm 0.12)\%,$$

(2.4)

The stochastic term is dominant for energies below about 2.5 GeV; above 2.5 GeV the constant term starts to be the limiting factor for the energy resolution.
2.6.2.2 Angular resolution

The angular resolution is determined by the transverse crystal size and the distance from the interaction point, and improves as the transverse size of the crystal decreases. On the other hand, since the electromagnetic shower has a natural lateral spread of the order of the Molière radius, the energy resolution would degrade if the transverse crystal size were chosen significantly smaller than the Molière radius, due to summing of the electronic noise from several crystals. The best compromise is obtained by choosing the transverse size of the crystals to be comparable to the Molière radius: this choice allows to achieve the required angular resolution (few milliradians) at low energies while maintaining the total number of crystals and readout channels limited to an acceptable noise and cost level.

Fig. 2-21(b) [57] shows the angular resolution measured as a function of energy. The decays of \( \pi^0 \) and \( \eta \) candidates in which the two photons in the decay have approximately equal energy are used to infer angular resolution. It varies between about 12 milliradians at low energies and 3 milliradians at high energy. The data fit the empirical parameterization:

\[
\sigma_{\theta,\phi} = \left( \frac{(3.87 \pm 0.07)}{\sqrt{E (\text{GeV})}} + (0.00 \pm 0.04) \right) \text{mrad}
\]

2.7 The Instrumented Flux Return

The Instrumented Flux Return (IFR) is designed to identify muons and neutral hadrons (primarily \( K_L \) and neutrons). Muons are important for tagging the flavor of neutral \( B \) mesons via semi-leptonic decays, for the reconstruction of vector mesons, like the \( J/\psi \), and the study of semi-leptonic and rare decays involving leptons from \( B \) and \( D \) mesons and \( \tau \) leptons. \( K_L \) detection allows for the study of exclusive \( B \) decays, in particular \( CP \) eigenstates; it could also help in vetoing charm decays and improve the reconstruction of neutrinos. The principal requirements for IFR are large solid angle coverage, good efficiency and high background rejection for muons down to momenta below 1 GeV/c. For neutral hadrons, high efficiency and good angular resolution are most important.

2.7.1 Detector layout

The IFR uses the steel flux return of the magnet as muon filter and hadron absorber. Single gap resistive plate chambers (RPC) with two-coordinate readout, operated in limited streamer mode constitute the active part of the detector [58]. The RPC are installed in the gaps of the finely segmented steel of the six barrel sectors and the two end-doors of the flux return, as illustrated in Fig. 2-22. The steel segmentation has been optimized on the basis of Monte Carlo studies of muon penetration and charged and neutral hadron interactions. The steel is segmented into 18 plates, increasing in thickness from 2 cm of the inner 9 plates to 10 cm of outermost plates for a total 65 cm. In addition, two layers of cylindrical RPCs are installed between the EMC and the magnet cryostat to detect particles exiting the EMC. Soon after the installation (which took place in Summer 1999), the efficiency of a significant fraction of the chambers (initially greater then 90%) has started to deteriorate at a rate of 0.5-1%/month. In order to solve some of the inefficiency problems
Figure 2-22. Overview of the IFR Barrel sectors and forward and backward end-doors; the shape of the RPC modules and the way they are stratified is shown.

an extensive improvement program has been developed and is making relevant advances. The RPCs in the forward end-cap region have been replaced in Summer 2002 with new ones based on the same base concept but with improved fabrication technique and quality controls: their efficiency has not significantly decreased over 2 years of running. The RPCs in the barrel region are being replaced with limited streamer tube (LST) detectors: two of the six sextants of the barrel have been replaced in Summer 2004 while the remaining four sextants should be replaced this year. Extensive quality control studies have been performed to check the reliability of these detectors, which are expected to operate until the end of the experiment with \( \approx 90\% \) efficiency, as measured in cosmic ray runs. RPCs detect streamers from ionizing particles via capacitive

Figure 2-23. Cross section of a planar RPC with the schematics of the HV connection.

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readout strips. They offer the advantage of simple and low cost construction. Further benefits are large signals and fast response allowing for simple and robust front end electronics and good time resolution, typically 1-2 ns. The position resolution depends on the segmentation of the readout; few millimeters are achievable. A cross section of an RPC is shown schematically in Figure 2-23. The planar RPCs consist of two bakelite sheets, 2 mm thick and separated by a gap of 2 mm. The bulk resistivity of the bakelite sheets has been especially tuned to $10^{11} \sim 10^{12} \Omega \text{cm}$. The external surfaces are coated with graphite to achieve a surface resistivity of approximately $100 \, k\Omega/cm^2$. These two graphite surfaces are connected to high voltage (approximately 8 kV) and ground, and protected by an insulating mylar film. The bakelite surfaces facing the gap are treated with linseed oil to improve performance. The modules are operated in limited streamer mode and the signals are read out capacitively, on both sides of the gap, by external electrodes made of aluminum strips on a mylar substrate.

2.7.2 Detector performance

2.7.2.1 RPCs efficiency

The efficiency of the RPCs is evaluated from samples of high momentum muons collected both in normal collision data (from the process $e^+e^- \rightarrow \mu^+\mu^-$) and dedicated cosmic ray runs. The efficiency is found by counting the number of times a hit found in a certain chamber when a charged track is expected, based on information from the other chambers and from the tracking system, to traverse it. The absolute efficiency at the nominal working voltage (typically 7.6 kV) is stored in the BABAR condition database for use in the event reconstruction software.

Following the installation and commissioning of the IFR system all the RPC modules were tested with cosmic rays and their average efficiency was measured to be approximately 92%.

Unfortunately, soon after installation, a progressive, steady efficiency deterioration has been observed in a significant fraction of the chambers. Detailed efficiency studies revealed large regions of very low efficiencies in the modules, but no clear pattern was identified. Tests to understand the efficiency decrease excluded several possible causes as the primary source of the problem, such as a change in the bakelite bulk resistivity, gas flow or composition. On the other hand, it was found that a number of prototype RPCs developed similar efficiency problems after being operated above a temperature $36^\circ C$ for a period of two weeks: in some of these modules evidence was found that the linseed oil had accumulated at various spots under the influence of the electric field.

In order to solve some of the inefficiency problems an extensive improvement program has been developed and is making relevant advances. The RPCs in the forward end-cap region have been replaced in Summer 2002 with new ones based on the same base concept but with an improved linseed oil esseraction technique: their efficiency has not significantly decreased over 2 years of running. The RPCs in the barrel region are being replaced with limited streamer tube (LST) detectors: two of the six sextants of the barrel have been replaced in Summer 2004 while the remaining four sextants will be replaced for the end of 2006. Extensive...

\footnote{Similar temperatures had been reached inside the iron during the first summer of operations due to the temperature in the experimental hall and the absence of a water cooling system}
quality control studies have been performed to check the reliability of these detectors, which are expected to operate until the end of the experiment with \( \approx 90\% \) efficiency, as measured in cosmic ray runs.

### 2.8 The \textit{BABAR} Trigger

The \textit{BABAR} trigger is designed to select a large variety of physics processes rejecting background events and keeping the total event rate under 280 Hz so as not to overload the downstream processing. The trigger must select the physics events of interest with very high and/or well understood efficiency, depending on the particular mode. Efficiency, diagnostic and background studies require the trigger to be able to select prescaled samples of Bhabha, di-muon and cosmic events. This kind of studies also demand random beam crossings and events that fail the trigger selection criteria.

The trigger system operates as a sequence of two independent stages, the second conditional upon the first. The Level 1 (L1) hardware trigger is performed first at the machine crossing rate. Its goal is to sufficiently reduce that rate to a level acceptable for the Level 3 (L3)\(^3\) software trigger which runs on a farm of commercial processors. The L1 trigger is optimized for simplicity and speed. It consists of a pipelined hardware processor. It is designed to provide an output trigger rate of the order of 2 kHz or less. The L1 trigger selection is based on a reduced data set from the DCH, EMC and IFR. Its maximum L1 response latency for a given collision is 12 \( \mu s \). Based on both the complete event and L1 trigger information, the L3 software algorithms select events of interest allowing them to be transferred to mass storage for further analysis. Dedicated L1 trigger processors receive data which is continuously clocked in from the DCH, EMC and IFR detector subsystems. The L1 trigger processor produces a 30 MHz clocked output to the Fast Control and Timing System (FCTS) that can optionally mask or prescale input triggers. The arrival of a L1-Accept signal by the data acquisition system causes a window of each subsystem’s L1 latency buffer to be read out.

The Level 3 trigger is implemented as a software that makes use of the complete event information for taking its decision, including the output of the L1 trigger processors and of the FCTS.

The selection decision is primarily taken by two set of orthogonal filters, one exclusively based on the DCH information, the other based on the EMC data only. The drift chamber filters select events containing at least one high \( p_T \) track (\( p_T > 600 \text{ MeV}/c \)) or two low \( p_T \) tracks, originating from the interaction point. The EMC filters look for events characterized by an effective mass greater than 1.5 GeV. The effective mass is calculated from the cluster energy sums and the energy weighted centroid positions of all clusters in the event in the massless particles hypothesis. The events must also contain at least two clusters with c.m. energy greater than 350 MeV or at least four clusters.

Table 2-6 shows the L3 and L1+L3 trigger efficiency for some relevant physics processes, derived from Monte Carlo simulation.

\(^3\)An intermediate Level 2 software trigger was originally foreseen in the very early step of \textit{BABAR} design, but it was soon merged in the L3 trigger

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<table>
<thead>
<tr>
<th>L3 Trigger</th>
<th>$\epsilon_{b\bar{b}}$</th>
<th>$\epsilon_{B\rightarrow\pi^+\pi^-}$</th>
<th>$\epsilon_{B\rightarrow\gamma\nu}$</th>
<th>$\epsilon_{c\bar{c}}$</th>
<th>$\epsilon_{uds}$</th>
<th>$\epsilon_{\gamma\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 track filter</td>
<td>89.9</td>
<td>69.9</td>
<td>86.5</td>
<td>89.2</td>
<td>88.2</td>
<td>94.1</td>
</tr>
<tr>
<td>2 track filter</td>
<td>98.9</td>
<td>84.1</td>
<td>94.5</td>
<td>96.1</td>
<td>93.2</td>
<td>87.6</td>
</tr>
<tr>
<td>Combined DCH filters</td>
<td>99.4</td>
<td>89.1</td>
<td>96.6</td>
<td>97.1</td>
<td>95.4</td>
<td>95.5</td>
</tr>
<tr>
<td>2 cluster filter</td>
<td>25.8</td>
<td>91.2</td>
<td>14.5</td>
<td>39.2</td>
<td>48.7</td>
<td>34.3</td>
</tr>
<tr>
<td>4 cluster filter</td>
<td>93.5</td>
<td>95.2</td>
<td>62.3</td>
<td>87.4</td>
<td>85.5</td>
<td>37.8</td>
</tr>
<tr>
<td>Combined EMC filters</td>
<td>93.5</td>
<td>95.5</td>
<td>62.3</td>
<td>87.4</td>
<td>85.6</td>
<td>46.3</td>
</tr>
<tr>
<td>Combined DCH+EMC filters</td>
<td>&gt;99.9</td>
<td>99.3</td>
<td>98.1</td>
<td>99.0</td>
<td>97.6</td>
<td>97.3</td>
</tr>
<tr>
<td>Combined L1+L3</td>
<td>&gt;99.9</td>
<td>99.1</td>
<td>97.8</td>
<td>98.9</td>
<td>95.8</td>
<td>92.0</td>
</tr>
</tbody>
</table>

Table 2-6. L3 trigger efficiency (%) for various physics processes, derived from Monte Carlo simulation.

2.9 Data sample

2.9.1 Data

The total dataset used in this analysis correspond to an integrated on-peak luminosity of 316.3 $fb^{-1}$, recorded by BABAR in the years 1999-2006. They correspond to about 347 million of $B \bar{B}$ pairs. The full dataset has been used to perform the analysis on the $B^+ \rightarrow \eta \ell^+\nu$ and $B^+ \rightarrow \eta' \ell^+\nu$ decay modes. Instead, for the analysis on the $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ modes, a subsample of data, corresponding to an integrated on-peak luminosity of 211 $fb^{-1}$, has been used. This subsample has been chosen to be consistent with the $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ analysis performed by BABAR using the semileptonic tag technique, in order to combine the measurements obtained by the two tagged methods (see Sec.7.4).

2.9.2 Monte Carlo samples

The Monte Carlo samples used in this analysis are summarized in Tab.2-7.

These samples differ either in terms of the decay modes of the fully reconstructed $B$ meson or in the selection of semileptonic decays and their decay model.

Cocktail samples contain specific hadronic decay modes for one of the $B$’s, corresponding to a subset of the modes used in the semi-exclusive reconstruction, described in Sec. 3.5, and where the reconstruction has very high efficiency. There are no requirement on the decay of the other $B$ meson. Cocktail samples have been used only for cross check purposes and high statistic tests.

Generic $b\bar{b}$ Monte Carlo represent the full simulation of all possible decays of the $B$ meson and it should represent the data and an unbiased event sample. This sample is actually used to model the data.
Table 2-7. Monte Carlo event samples used in this analysis.

<table>
<thead>
<tr>
<th>Data Set (mode #)</th>
<th>1* B mode</th>
<th>2* B mode</th>
<th>equiv. lumin.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$ Cocktail</td>
<td>Generic</td>
<td>Cocktail</td>
<td>$\sim 2400 , fb^{-1}$</td>
</tr>
<tr>
<td>$B^{\pm}$ Cocktail</td>
<td>Generic</td>
<td>Cocktail</td>
<td>$\sim 840 , fb^{-1}$</td>
</tr>
<tr>
<td>$B^0$ Generic</td>
<td>Generic</td>
<td>Generic</td>
<td>$\sim 996 , fb^{-1}$</td>
</tr>
<tr>
<td>$B^{\pm}$ Generic</td>
<td>Generic</td>
<td>Generic</td>
<td>$\sim 1033 , fb^{-1}$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$ pure res. Generic</td>
<td>$b \to u\ell\nu$ exclusive</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$ pure res. Cocktail</td>
<td>$b \to u\ell\nu$ exclusive</td>
</tr>
</tbody>
</table>

Signal Monte Carlo samples contain only pure resonant exclusive $B \to X_u\ell\nu$ decays, based on the measured values reported in [59] and on the theoretical expectation. The ISGW2 theoretical model [17] is used to generate these samples.

Usage of Monte Carlo samples in this analysis is summarized in Tab.2-8.

Table 2-8. Usage of Monte Carlo samples in this analysis.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$, $B^{\pm}$ cocktail</td>
<td>selection criteria optimization</td>
</tr>
<tr>
<td>$B^0$, $B^{\pm}$ generic</td>
<td>$b \to c\ell\nu$ background modeling in all data fits; plots of background components;</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
</tr>
</tbody>
</table>

Semileptonic $B$ decays in the generic Monte Carlo simulation have been modeled with specific decay models for the charm meson. A parameterization of HQET form factors, defined in [60], is used for $B \to D^{(*)}\ell\nu$, the model of Goity and Roberts [61] is used for the non-resonant decays $B \to D^{(*)}\pi\ell\nu$, whereas the ISGW2 model [17] is used for all other semileptonic decays.

The non $b\bar{b}$ Monte Carlo sample consists of $c\bar{c}$ and $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ events. These samples have been used, as the off-peak data, to model the background (see Sec.6.2.2)
Event Reconstruction

As already mentioned in the introduction, the analysis is based on the study of the recoil of $B$ mesons fully reconstructed in an hadronic mode ($B_{\text{reco}}$).

The use of this method offers many advantages. First of all it assures a very clean environment to study the properties of the recoil. One of the two $B$ mesons from the decay of the $\Upsilon (4S)$ is reconstructed in a fully hadronic mode (see Fig.3-1). Then, the remaining particles of the event are then supposed to belong to the other $B$ ($B_{\text{recoil}}$).

In case of semileptonic decay of the recoiling $B$ the only missing particle is a neutrino. This implies that the charge conservation of the event can be applied and that the missing mass of the event should be compatible with zero. Moreover, since the kinematics is over-constrained, the resolution on the reconstructed quantities, such as the mass $m_X$ of the hadronic system $X$, is improved.

The momentum of the recoiling $B$ is also known and therefore it’s possible to apply a Lorentz transformation to the charged lepton four-momentum and compute it in the $B$ rest frame. The charge of the $B$ is known, so $B^0$ and $B^+$ decays can be studied separately. The flavor of the $B$ is known, therefore the correlation between the charge of the lepton and the flavor of the $B$ can be used to reject $B \to D \to \text{lepton}$ cascade events, as described in Sec.4.3.1.

The only drawback is that the efficiency of this method is quite low and it is dominated by the $B$ reconstruction efficiency that is $\sim 0.4\%$. In the analysis all the efficiencies will be calculated with respect to a sample with at least one fully reconstructed $B$ and and a charged lepton in the recoil. The charged and neutral particles reconstruction is described in this chapter together with the algorithm used for the full hadronic $B$ reconstruction, the so-called Semi-exclusive reconstruction [62] (Sec. 3.5).

3.1 Charged particles reconstruction

The charged particle tracks are reconstructed by processing the information from both tracking systems, the SVT and the DCH. Charged tracks are defined by five parameters ($d_0, \phi_0, \omega, z_0, \tan \lambda$) and their associated error matrix, measured at the point of closest approach to the z-axis. $d_0$ and $z_0$ are the distances between the point and the origin of the coordinate system in the x-y plane and along the z-axis respectively. The angle $\phi_0$ is the azimuth of the track, $\lambda$ is the angle between the transverse plane and the track tangent vector at the point of closest approach and the x-axis, and $\omega = 1/p_t$ is the curvature of the track. $d_0$ and $\omega$ are signed variables and their sign depends on the charge of the track. The track finding and the fitting procedures use the Kalman filter algorithm [63] that takes into account the detailed distribution of material in the detector and the full magnetic field map.
Figure 3-1. *Semileptonic events on the recoil of a fully reconstructed B meson.*

For what concerns this analysis, the definition of charged track is based on some specific quantities:

- **Distance of closest approach to the beam spot** measured in the $x$ - $y$ plane ($|d_{xy}|$) and along the $z$ axis ($|d_z|$). A cut on those variables rejects fake tracks and background tracks not originating near the beam-beam interaction point. We require $|d_{xy}| < 1.5$ cm and $|d_z| < 10$ cm.

- **Number of associated hits in DCH.** For high momentum tracks ($p_{T} > 0.2$ GeV/$c$) we require a minimum number of associated hits in the DCH. This request is not applied to low momentum tracks since slow pions, produced for instance in the $Q_{SRTUVQXWZY}$ decays, would be rejected.

- **Maximum momentum.** To remove tracks not compatible with the beam energy we require $p_{lab} < 10$ GeV/$c$, where $p_{lab}$ refers to the laboratory momentum of the track, against misreconstructed tracks.

- **Polar angle acceptance.** The polar angle, in the laboratory frame, is required to be $0.41 < \theta_{lab} < 2.54$ in order to match the acceptance of the detector. This ensures a well-understood tracking efficiency and systematics.

No restrictions on the impact parameter have been imposed for secondary tracks from $K_s$ decays. No cut on the minimum number of hits on track is used in order to maximize the efficiency for low momenta tracks.

In addition, special criteria are used to reject tracks due to specified tracking errors. Tracks with a transverse momentum $p_{T} < 0.18$ GeV/$c$ don’t reach the EMC and therefore they will spiral inside the DCH (“loopers”). The tracking algorithms of BABAR will not combine the different fragments of these tracks into a single track. Therefore dedicated cuts have been developed to reject track fragments compatible with originating from looper based on their distance from the beam spot. Looper candidates are identified as two tracks with a small difference in $p_{T}$, $\phi$ and $\theta$. Of such a pair only the track fragments with the smallest distance $|d_z|$ to the beam interaction point is retained. These cuts remove roughly 13% of all low-momentum tracks in the central part of the detector. On average, the mean observed charged multiplicity per $B$ meson is reduced by less than 1%.

Two tracks very closely aligned to each other are called ”ghosts”. These cases arise when the tracking algorithms splits the DCH hits belonging to a single track in two track fragments. If two tracks are very close in phase space only the track with the largest number of DCH hits is retained. This ensures that the fragment with the better momentum measurement is kept in the analysis.

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3.2 Particle identification

3.2.1 Electron identification

The EMC is crucial for electron identification. Different criteria are established to select electrons with different level of purity and efficiency. Electrons are primarily separated from charged hadrons by taking into account the ratio of the energy $E$ deposited in the EMC to the track momentum $p$ ($\frac{E}{p}$). This quantity should be compatible with the unity for electrons, since all their energy is deposited in the calorimeter. The other charged tracks should appear as minimal ionizing particles, unless they have hadronic interactions in the calorimeter crystals. To further separate hadrons a variable describing the shape of the energy deposition in the EMC ($LAT$) is used. In addition, the $dE/dx$ energy loss in the DCH and the DIRC Čerenkov angle are required to be consistent with the values expected for an electron. This offers a good $e/\pi$ separation in a wide range.

The track selection criteria are tightened for electrons selection to suppress background and to ensure a reliable momentum measurement and identification efficiency. There are requirements in addition for transverse momentum $p_\perp > 0.1 \text{ GeV}/c$, and $N_{DCH} \geq 12$ for the number of associated drift chamber hits. Furthermore, only electron candidates with a laboratory momentum $p_{lab} > 0.5 \text{ GeV}/c$ are considered.

Electrons are identified using the a likelihood-based selector [64], which uses a number of discriminating variables.
- $E_{\text{cal}}/p_{\text{lab}}$, the ratio of $E_{\text{cal}}$, the energy deposited in the EMC, and $p_{\text{lab}}$ the momentum in the laboratory rest frame measured using the tracking system; $LAT$, the lateral shape of the calorimeter deposit (defined by eq. 3.2); $\Delta \Phi$, the azimuthal distance between the centroid of the EMC cluster and the impact point of the track on the EMC; and $N_{\text{cry}}$, the number of crystals in the EMC cluster;
- $dE/dx$, the specific energy loss in the DCH;
- the Čerenkov angle $\theta_C$ and $N_C$, the number of photons measured in the DRC.

First, muons are rejected on the basis of $dE/dx$ ratio value and the shower energy relative to the momentum. For the remaining tracks, likelihood functions are computed assuming the particle is an electron, pion, kaon, or proton. These likelihood functions are based on probability density functions that are derived from pure particle data control samples for each of the discriminating variables. For hadrons, we take into account the correlations between energy and shower-shapes. Using combined likelihood functions

$$L(\xi) = P(E/p, LAT, \Delta \Phi, dE/dx, \theta_C|\xi)$$

$$= P_{\text{Emc}}(E/p, LAT, \Delta \Phi|\xi) \times P_{\text{Dch}}(dE/dx|\xi) \times P_{\text{DRC}}(\theta_C|\xi)$$

for the hypotheses $\xi \in \{e, \pi, K, p\}$, the fraction

$$F_\theta = \frac{f_\theta L(e)}{\sum_\xi f_\xi L(\xi)}, \quad (3.1)$$

is defined, where, for the relative particle fractions, $f_\theta : f_\pi : f_K : f_p = 1 : 5 : 1 : 0.1$ is assumed. A track is identified as an electron if $F_\theta > 0.95$.

The electron identification efficiency has been measured using radiative Bhabha events, as function of laboratory momentum $p_{\text{lab}}$ and polar angle $\theta_{\text{lab}}$. The misidentification rates for pions, kaons, and protons are extracted from selected data samples. Pure pions are obtained from kinematically selected $K^0 \rightarrow \pi^+\pi^-$ decays and three prong $\tau^\pm$ decays. Two-body $\Lambda$ and $D^0$ decays provide pure samples of protons and charged kaons.

The performance of the likelihood-based electron identification algorithm is summarized in Fig. 3-2, in terms of the electron identification efficiency and the per track probability that an hadron is misidentified as an electron.

The average hadron fake rates per track are determined separately for positive and negative particles, taking into account the relative abundance from Monte Carlo simulation of $B \bar{B}$ events, with relative systematic uncertainties of 3.5%, 15% and 20% for pions, kaons, and protons, respectively. The resulting average fake rate per hadron track of $p_{\text{lab}} > 1.0 \text{ GeV}/c$, is of the order of 0.05% for pions and 0.2% for kaons.

### 3.2.2 Muon identification

Muons are identified by measuring the number of traversed interaction lengths in the entire detector and comparing it with the number of expected interaction lengths predicted for a muon of the same momentum.

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Moreover, the projected intersections of a track with RPC or LST planes are computed and, for each readout plane, all clusters (groups of adjacent hits in one of the two readout coordinates) detected within a maximum distance from the predicted intersection are associated with the track. An additional $\pi/\mu$ discriminating power is provided by the average number and the r.m.s. of the distribution of the RPC and LST hits per layer. The average number of hits per layer is expected to be larger for pions, producing an hadronic interaction, than for muons. Other variables exploiting clusters distribution shapes are constructed and criteria, based on all these variables, are applied to select muons. The muon selection performance has been tested on samples of kinematically identified muons from $\mu\mu e e$ and $\mu\mu \gamma$ final states and pions from three-prong $\tau$ decays and $K_S^0$ decays.

The muon selection procedure is as follows:

- tight criteria on tracking: $p_L > 0.1 \text{ GeV}/c$, $N_{DCH} \geq 12$, $0.360 < \theta_{lab} < 2.37$ and $p_{lab} > 1.0 \text{ GeV}/c$
- the energy deposited in the EMC is required to be consistent with the minimum ionizing particle hypothesis, $50 \text{ MeV} < E_{cal} < 400 \text{ MeV}$;
- the number of IFR layers associated with the track has to be $N_L \geq 2$.
- the interaction lengths of material traversed by the track has to be $\lambda_{meas} > 2.2$.
- The number of interaction lengths expected for a muon of the measured momentum and angle to traverse is estimated by extrapolating the track up to the last active layer of the IFR. This estimate takes into account the IFR efficiencies which are routinely measured and stored. The difference $\Delta \lambda = \lambda_{exp} - \lambda_{meas}$ is required to be $< 1.0$ for tracks with momentum greater than $1.2 \text{ GeV}/c$. For track momenta between $0.5 \text{ GeV}/c$ and $1.2 \text{ GeV}/c$, a variable limit is placed: $\Delta \lambda < [(p_{lab} - 0.5)/0.7]$.
- The continuity of the IFR cluster is defined as $T_c = \frac{N_L}{L + F + T}$, where $L$ and $F$ are the last and first layers with hits. $T_c$ is expected to be 1.0 for muons penetrating an ideal detector whereas is expected smaller.

**Event Reconstruction**
for hadrons. We require $T_e > 0.3$ for tracks with $0.3 < \theta_{lab} < 1.0$ (i.e. in the Forward End Cap to remove beam background).

- The observed number of hit strips in each RPC or LST layer is used to impose the conditions on the average number of hits, $\bar{m} < 8$, and the standard deviation, $\sigma_m < 4$.
- The strip clusters in the IFR layers are combined to form a track and fit to a third degree polynomial, with the quality of the fit selected by the condition $\chi^2_{fit}/DOF < 3$. In addition, the cluster centroids are compared to the extrapolated charged track, with the requirement $\chi^2_{trk}/DOF < 5$.

The muon identification efficiency has been measured using $\mu^+\mu^-(\gamma)$ events and two-photon production of $\mu^+\mu^-$ pairs. The misidentification rates for pions, kaons, and protons are extracted from selected data samples. The performance of the muon identification algorithm is summarized in Fig. 3-3, in terms of the muon identification efficiency. The errors shown are statistical only, the systematic error is dominated by variations in the performance of the IFR as a function of position and time.

### 3.2.3 Charged kaon identification

A standard selector, based only on track candidates with an associated momentum above 300 GeV/c and exploiting variables based on information from the DRC, the DCH and the SVT, is used to identify charged

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Neutral particles reconstruction

Neutral particles (photons, $\pi^0$, neutral hadrons) are detected in the EMC as clusters of close crystals where energy has been deposited. They are required not to be matched to any charged track extrapolated from the tracking volume to the inner surface of the EMC.

For this analysis a neutral particle is selected by its local maximum energy depositions in the EMC. These energy clusters originate mostly from photons, thus momenta and angles are assigned to be consistent with photons originating from the interaction region. The list of neutrals is also used to reconstruct the neutral pions. In Sec.3.4.1 and 4.2.1 are described the selection of the $\pi^0$ candidates used in the $B_{\text{reco}}$ and $B_{\text{recoll}}$ reconstruction, respectively.

Photon candidates are required to have an energy $E_{\gamma} > 30\text{ MeV}$ in order to reduce the impact of the sizable beam-related background of low energy photons. Some additional backgrounds, due to hadronic interactions, either by $K_L$ or neutrons, can be reduced by applying requests on the shape of the calorimeter clusters.

The variable $LAT$, used to discriminate between electromagnetic and hadronic showers in the EMC, is defined as

$$LAT = \frac{\sum_{i=3}^{N} E_i r_i^2}{\sum_{i=3}^{N} E_i r_i^2 + E_1 r_1^2 + E_2 r_2^2},$$

(3.2)
where \( N \) is the number of crystals associated with the electromagnetic shower, \( r_0 \) is the average distance between two crystals, which is approximately 5 cm for the \( \text{BABAR} \) calorimeter. \( E_i \) is the energy deposited in the \( i \)-th crystal, numbering them such that \( E_1 > E_2 > \ldots > E_N \) and \( r_i, \phi_i \) are the polar coordinates in the plane perpendicular to the line pointing from the interaction point to the shower center centered in the cluster centroid. Considering that the summations start from \( i = 3 \), they omit the two crystals containing the highest amounts of energy. Since electrons and photons deposit most of their energy in two or three crystals, the value of LAT is small for electromagnetic showers. Multiplying the energies by the squared distances enhances the effect for hadronic showers, compared with electromagnetics ones.

Another useful shape variable is the so-called \( S9S25 \), that is the ratio of the energy deposited in the 9 closest crystals from the cluster centroid over the energy deposited in the 25 closest clusters. Are assigned to the neutral particles all the local energy maxima, not matching with charged tracks, and store in a list the relative parameters, determined by assuming that the particle is a photon.

Clusters, which are considered as neutral candidates, although they are close to tracks, need to be rejected. These unmatched clusters are due to inefficiencies in the matching algorithms and lead to double counting of their energies.

For this purpose the angle between the positions of the cluster and the impact point of the nearest charged track at the EMC surface is considered. The 3-D angular difference \( \Delta \alpha \) is given by:

\[
\Delta \alpha = \cos^{-1} \left[ \cos \theta_{ct} \cos \theta_{tr} + \sin \theta_{ct} \sin \theta_{tr} \cos (\phi_{ct} - \phi_{tr}) \right]
\]  

where \( \theta_{ct, tr} \) and \( \phi_{ct, tr} \) are the polar coordinates for clusters and tracks respectively. \( \Delta \alpha \) is required to be > 0.08 for charged tracks that do not pass the electron likelihood selector. Any request is made on this variable for electrons.

Assuming that the neutral detected particle is a photon, two different neutral lists, distinguished by reconstruction criteria, have been defined:

- a loose photon list is obtained using neutral candidates composed by single EMC bumps not matched to any track (the photon mass hypothesis is assigned). The energy in the laboratory rest frame \( E_\gamma > 30 \text{MeV} \) and the Lateral Moment of the energy distribution, \( LAT < 0.8 \) are required.

- a tight photon list is obtained refining the loose list described above by applying additional quality criteria, in order to reject combinatorics. We use here only photons with an energy in the laboratory \( E_\gamma > 80 \text{MeV} \) and in addition \( S9S25 > 0.9 \) and \( \Delta \alpha > 0.08 \) are required.

Tab. 3-2 shows a detailed summary of the selection criteria applied on the neutral candidates.

### 3.4 Meson Reconstruction

This section describes the reconstruction of the mesons used in the full reconstruction of the \( B \). Different control samples have been used to produce the following plots and the statistics do not correspond to that of the final sample.

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3.4 Meson Reconstruction

<table>
<thead>
<tr>
<th></th>
<th>Selection criteria for loose list</th>
<th>Selection criteria for tight list</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral energy</td>
<td>$E_\gamma &gt; 30$ MeV</td>
<td>$E_\gamma &gt; 80$ MeV</td>
</tr>
<tr>
<td>LAT</td>
<td>LAT &lt; 0.8</td>
<td>LAT &lt; 0.8</td>
</tr>
<tr>
<td>$S9S25$</td>
<td>-</td>
<td>$S9S25 &gt; 0.9$</td>
</tr>
<tr>
<td>unmatched clusters</td>
<td>-</td>
<td>$\Delta \alpha &gt; 0.08$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>from nearest track (not electrons)</td>
</tr>
</tbody>
</table>

Table 3-2. Summary of the photon selections.

### 3.4.1 $\pi^0$ reconstruction

A wide energy spectrum of $\pi^0$'s ranging from particles almost at rest up to several GeV is needed in this analysis. For instance, lowest energy $\pi^0$'s are used to reconstruct the $D^{*0} \rightarrow D^0 \pi^0$ decays while the decay products in the $B \rightarrow D \pi \pi^0$ channel have quite large momentum.

The $\pi^0$'s are reconstructed using pairs of neutral clusters with a lower energy at 30 MeV and applying a cut on the LAT variable. The $\pi^0$ candidate has to have an energy above 200 MeV. A mass windows of 110-155 MeV/c^2, corresponding to $(-4\sigma, +3\sigma)$, is required. In Fig.3-5 the invariant masses distributions for simulated events and data are shown.

![Figure 3-5](image) Distribution of the invariant mass of the $\pi^0$ candidates for simulated events and for data.
3.4.2 $K_s^0$ reconstruction

$K_s^0$ are reconstructed in the channel $K_s^0 \to \pi^+ \pi^-$ by pairing all possible tracks of opposite sign and looking for the 3D point (vertex) which is more likely to be common to the two tracks. The algorithm is based on a $\chi^2$ minimization and uses as starting point for the vertex finding the point of closest approach of the two tracks in 3D. No constraint is applied on the invariant mass of the pair, but a $\pm 3\sigma$ cut around the nominal value is imposed: $0.486 < m_{\pi^+ \pi^-} < 0.510 \text{GeV}/c^2$. The invariant mass distribution of the $\pi^+ \pi^-$ obtained from data is shown in Fig.3-6. A comparison between data and Monte Carlo for the $K_s^0$ momentum and polar angle is shown in Fig.3-7. The channel $K_s^0 \to \pi^0 \pi^0$ is not used in this analysis.

![Figure 3-6](image)

**Figure 3-6.** Mass distributions for $K_s^0 \to \pi^+ \pi^-$ on data. The distribution is fitted with a sum of a double Gaussian and a first order polynomial function.

![Figure 3-7](image)

**Figure 3-7.** $K_s^0$ momentum (left) and polar angle (right) distributions in data (solid markers) and Monte Carlo simulation (hatched histogram), normalized to the same area.
### 3.4.3 $D$ reconstruction

The reconstruction of the $B$ mesons in hadronic modes utilizes charmed $D$ mesons decaying in a variety of channels. These channels and their branching fractions are summarized in Tab. 3-3.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching Fraction(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^* \to D^0 \pi; D^0 \to K\pi$</td>
<td>$2.55 \pm 0.06$</td>
</tr>
<tr>
<td>$D^* \to D^0 \pi; D^0 \to K\pi\pi^0$</td>
<td>$8.8 \pm 0.6$</td>
</tr>
<tr>
<td>$D^* \to D^0 \pi; D^0 \to K\pi\pi^0 (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$1.35 \pm 0.08$</td>
</tr>
<tr>
<td>$D^+ \to K\pi\pi^0$</td>
<td>$9.1 \pm 0.6$</td>
</tr>
<tr>
<td>$D^+ \to K_S^0 \pi (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$0.94 \pm 0.06$</td>
</tr>
<tr>
<td>$D^+ \to K\pi\pi^0$</td>
<td>$6.4 \pm 1.1$</td>
</tr>
<tr>
<td>$D^+ \to K_S^0 \pi\pi\pi (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$2.38 \pm 0.31$</td>
</tr>
<tr>
<td>$D^+ \to K_S^0 \pi\pi\pi (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$3.5 \pm 1.0$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \pi^0; D^0 \to K\pi$</td>
<td>$2.35 \pm 0.12$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \pi^0; D^0 \to K3\pi$</td>
<td>$4.6 \pm 0.3$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \pi^0; D^0 \to K\pi\pi^0$</td>
<td>$8.1 \pm 0.7$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \pi^0; D^0 \to K_S^0 \pi\pi (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$1.2 \pm 0.1$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \gamma; D^0 \to K\pi$</td>
<td>$1.44 \pm 0.19$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \gamma; D^0 \to K3\pi$</td>
<td>$2.82 \pm 0.18$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \gamma; D^0 \to K\pi\pi^0$</td>
<td>$5.0 \pm 0.4$</td>
</tr>
<tr>
<td>$D_s^0 \to D^0 \gamma; D^0 \to K_S^0 \pi\pi (K_S^0 \to \pi^+ \pi^-)$</td>
<td>$0.7 \pm 0.1$</td>
</tr>
<tr>
<td>$D^0 \to K\pi$</td>
<td>$3.80 \pm 0.09$</td>
</tr>
<tr>
<td>$D^0 \to K3\pi$</td>
<td>$7.46 \pm 0.31$</td>
</tr>
<tr>
<td>$D^0 \to K\pi\pi^0$</td>
<td>$14.0 \pm 0.9$</td>
</tr>
<tr>
<td>$D^0 \to K_S^0 \pi\pi$</td>
<td>$2.03 \pm 0.12$</td>
</tr>
</tbody>
</table>

**Table 3-3.** $D$ mesons decays used in the Semi-exclusive $B$ reconstruction.

### 3.4.3.1 $D^0$ selection

The $D^0$ is reconstructed in the modes $D^0 \to K\pi$, $D^0 \to K3\pi$, $D^0 \to K\pi\pi^0$ and $D^0 \to K_S^0 \pi\pi$. The charged tracks originating from a $D$ meson are required to have a minimum momentum of 200 MeV/c for the channel $D^0 \to K\pi$ and 150 MeV/c for the remaining three modes. The $D^0$ candidates are required to lie within $\pm 3\sigma$ of the nominal $D^0$ mass. All $D^0$ candidates must have momentum greater than 1.3 GeV/c and lower than 2.5 GeV/c in the $\Upsilon(4S)$ frame. The lower bound is needed to reduce combinatorics,
the upper one is the kinematic endpoint of the $D^0$ coming from a $B \to D^0 X$ decay or $B \to D^{*+} X$ with $D^{**} \to D^0 \pi^+$. A vertex fit is performed and a $\chi^2$ probability greater than 0.1% is required. The selection criteria are summarized in Tab. 3-4.

<table>
<thead>
<tr>
<th></th>
<th>$D^0 \to K \pi$</th>
<th>$D^0 \to K \pi \pi^0$</th>
<th>$D^0 \to K3\pi$</th>
<th>$D^0 \to K_s^0 \pi \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_D$ invariant mass window</td>
<td>$\pm 15$ MeV/c$^2$</td>
<td>$\pm 25$ MeV/c$^2$</td>
<td>$\pm 15$ MeV/c$^2$</td>
<td>$\pm 20$ MeV/c$^2$</td>
</tr>
<tr>
<td>Charged Tracks : lower $p^*$ cut</td>
<td>$&gt; 200$ MeV/c</td>
<td>$&gt; 150$ MeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D^0$ upper $p^*$ cut</td>
<td></td>
<td></td>
<td>$&lt; 2.5$ GeV/c</td>
<td></td>
</tr>
<tr>
<td>$D^0$ lower $p^*$ cut</td>
<td></td>
<td></td>
<td>$&gt; 1.3$ GeV/c</td>
<td></td>
</tr>
<tr>
<td>Vertex Fit</td>
<td></td>
<td></td>
<td></td>
<td>$\chi^2 &gt; 0.01$</td>
</tr>
</tbody>
</table>

Table 3-4. Summary of criteria applied for the $D^0$ selection.

3.4.3.2 $D^+$ selection

$D^+$ candidates are reconstructed in the modes $D^+ \to K^- \pi^+ \pi^+ \pi^+$, $D^+ \to K^- \pi^+ \pi^+ \pi^0$, $D^+ \to K_s^0 \pi^+ \pi^+$, $D^+ \to K_s^0 \pi^+ \pi^0$, $D^+ \to K_s^0 \pi^+ \pi^+ \pi^+$. We require that the kaon used in the $K^- \pi^+ \pi^+$ and $K^- \pi^+ \pi^+ \pi^0$ modes have a minimum momentum of 200 MeV/c while the pions are required to have momentum greater than 150 MeV/c. For the $K_s^0 \pi^+ X$ modes, the minimum charged track momentum is required to be 200 MeV/c. $D^+$ candidates are required to have an invariant mass within $\pm 3\sigma$, calculated on an event-by-event basis, of the nominal $D^+$ mass. The $D^+$ candidates must have momentum greater than 1.0 GeV/c in the $\Upsilon(4S)$ frame for the three cleanest modes ($D^+ \to K^- \pi^+ \pi^+$, $D^+ \to K_s^0 \pi^+$ and $D^+ \to K_s^0 \pi^+ \pi^0$) and greater than 1.6 GeV/c for the two remaining ones ($D^+ \to K^- \pi^+ \pi^+ \pi^0$ and $D^+ \to K_s^0 \pi^+ \pi^+ \pi^+$). Moreover, all $D^+$ candidates must have momentum lower than 2.5 GeV/c in the $\Upsilon(4S)$ frame, as the $D^0$ case. A vertex fit is performed and a $\chi^2$ probability greater than 0.1% is required. The selection criteria are summarized in Tab. 3-5.

<table>
<thead>
<tr>
<th></th>
<th>$D^+ \to K \pi \pi$</th>
<th>$D^+ \to K_s^0 \pi$</th>
<th>$D^+ \to K_s^0 \pi \pi^0$</th>
<th>$D^+ \to K_s^0 \pi \pi \pi^0$</th>
<th>$D^+ \to K_s^0 \pi \pi \pi \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{D \pi}$ invariant mass window</td>
<td>$\pm 20$ MeV/c$^2$</td>
<td>$\pm 20$ MeV/c$^2$</td>
<td>$\pm 30$ MeV/c$^2$</td>
<td>$\pm 30$ MeV/c$^2$</td>
<td>$\pm 30$ MeV/c$^2$</td>
</tr>
<tr>
<td>$D^+ \text{ lower } p^*$ cut</td>
<td></td>
<td>$&gt; 1.0$ GeV/c</td>
<td></td>
<td>$&gt; 1.6$ GeV/c</td>
<td></td>
</tr>
<tr>
<td>$D^+ \text{ upper } p^*$ cut</td>
<td></td>
<td></td>
<td>$&lt; 2.5$ GeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charged Tracks : lower $p^*$ cut</td>
<td></td>
<td></td>
<td>$&gt; 200$ MeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex fit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\chi^2 &gt; 0.01$</td>
</tr>
</tbody>
</table>

Table 3-5. Summary of criteria applied for the $D^+$ selection.
3.4 Meson Reconstruction

![Figure 3-8](image)

**Figure 3-8.** Distribution of soft pion momentum in the $\Upsilon(4S)$ frame (left) and $m(D^{*+}\pi^-) - m(D^0)$ mass distribution for $D^{*+}$ candidates in the $B \rightarrow D^{*+}\pi^- , D^0 \rightarrow K\pi$ mode. Units in both plots are GeV. Vertical lines indicate the signal windows used in the selection.

### 3.4.3.3 $D^{*+}$ selection

$D^{*+}$ candidates are formed by combining a $D^0$ with a pion which has momentum greater than 70 MeV/c. Due to the limited phase space available from the $D^* - D^0$ mass difference $\Delta m$, the pion coming from the $D^*$ has a low momentum, below 450 MeV/c, and is referred to as soft pion (see Fig.??). Only the reconstruction of the $D^{*+} \rightarrow D^0 \pi^+$ channel is discussed here, since $D^{*+} \rightarrow D^+ \pi^0$ events enter in the $B \rightarrow D^+ X$ category of the Semi-exclusive reconstruction, as explained in Sec.3.5. A vertex fit for the $D^{*+}$ is performed using a constraint to the beam spot to improve the angular resolution for the soft pion. A fixed $\sigma_y = 30 \mu$m is used to model the beam spot spread in the vertical direction, to avoid bias in the $D^{*+}$ vertex fit. The fit is required to converge, but no cut is applied on the probability of $\chi^2$. After fitting, selected $D^{*+}$ candidates are required to have $\Delta m$ within $\pm 3\sigma$ of the measured nominal value (see Fig.3-8). $\Delta m$ distribution is fitted with a double Gaussian distribution. The width is taken to be a weighted average of the core and broad Gaussian distributions. The selection criteria are summarized in Tab. 3-6.

### 3.4.3.4 $D^{*0}$ selection

$D^{*0}$ candidates are reconstructed by combining a selected $D^0$ with a either a $\pi^0$ or a photon having a momentum less than 450 MeV/c in the $\Upsilon(4S)$ frame. The minimum momentum for the $\pi^0$ corresponds to 70 MeV while the photons are required to have an energy greater than 100 MeV. For $D^{*0} \rightarrow D^0 \pi^0$ decay, selected $D^{*0}$ candidates are required to have $\Delta m$ within 4 MeV/c$^2$ of the nominal value while a
### Table 3-6. Summary of criteria applied for the $D^{*+}$ selection

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{*+} \rightarrow D^0 \pi^+$</td>
<td></td>
</tr>
<tr>
<td>Vertexing and $\chi^2$</td>
<td></td>
</tr>
<tr>
<td>$m(D^0 \pi^+) - m(D^0)$</td>
<td>$\pm 3\sigma$ MeV/$c^2$</td>
</tr>
<tr>
<td>$p^*(\pi^+)$</td>
<td>$[70, 450]$ MeV/$c$</td>
</tr>
</tbody>
</table>

wider window, $127 \text{ MeV}/c^2 < \Delta m < 157 \text{ MeV}/c^2$, is used for $D^{*0} \rightarrow D^0 \gamma$. The selection criteria are summarized in Tab. 3-7.

### Table 3-7. Summary of criteria applied for the $D^{*0}$ selection

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{*0} \rightarrow D^0 \pi^0$</td>
<td></td>
</tr>
<tr>
<td>$m(D^0 \pi^0) - m(D^0)$</td>
<td>$\pm 4$ MeV/$c^2$</td>
</tr>
<tr>
<td>$p^*(\pi^0)$</td>
<td>$[70, 450]$ MeV/$c$</td>
</tr>
<tr>
<td>$p^*(D^{*0})$</td>
<td>$1.3 &lt; p^* &lt; 2.5$ GeV/$c$</td>
</tr>
</tbody>
</table>

### 3.5 Semi-exclusive Reconstruction Method

The aim of the Semi-exclusive reconstruction is to get as many as possible $B$ mesons in fully hadronic modes in order to study the properties of the recoiling $B$ meson.

The sum of a few, very pure exclusive modes ensures very high purity but low efficiency. On the other hand a fully inclusive approach with high multiplicities is not feasible since the level of combinatoric background would be too high. A compromise has been set up, where only favored modes are considered and an algorithm which combines the final state particles neglecting the intermediate states as inclusive as possible is used.

Since $B^0$ mesons mostly decay into charged $D^{(*)}$ mesons while $B^-$ mesons decay into the neutral $D^{0(*)}$ mesons, only $B^- \rightarrow D^{0(*)}_y$, $B^0 \rightarrow D^{*+(*)}_y$ modes are considered. Tab. 3-8 shows the relevant branching fractions for $B$ mesons decaying predominantly into fully hadronic final states.

The Semi-exclusive reconstruction approach comprises the following steps:

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3.5 Semi-exclusive Reconstruction Method

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D^* \pm Y$</td>
<td>22.5 ± 1.5</td>
</tr>
<tr>
<td>$B \to D^{\pm} Y$</td>
<td>23.5 ± 1.9</td>
</tr>
<tr>
<td>$B \to D^{\ast 0} / D^{0} \ Y$</td>
<td>26.0 ± 2.7</td>
</tr>
<tr>
<td>$B \to D^0 / D^{\ast 0} \ Y$</td>
<td>64.0 ± 3.0</td>
</tr>
<tr>
<td>$B \to D^{\ast} Y$</td>
<td>10.5 ± 2.6</td>
</tr>
<tr>
<td>$B \to D^{-(<em>)} D_{K}^{(</em>)}$</td>
<td>4.9 ± 1.2</td>
</tr>
<tr>
<td>$B \to D^{-(<em>)} D^{(</em>)} K$</td>
<td>7.1 ± 2.3</td>
</tr>
<tr>
<td>$B^0 \to D^{-(<em>)} D^{(</em>)}$</td>
<td>~ 1.0</td>
</tr>
</tbody>
</table>

Table 3-8. Inclusive and Exclusive branching fractions relevant to this analysis as measured in [59].

- reconstruct all possible decay modes $B \to D Y$, where the $Y$ system is a combination of $\pi^+, \pi^0, \ K^+$ and $K^0_S$, with total charge equal to ±1, and including a maximum of 7 particles, 5 charged tracks and 2 neutrals;

- study the structure of the $Y$ system looking for resonances in the signal and studying the shape of the background (Sec.3.5.2);

- identify submodes and create sub-categories according to the their multiplicity and to the structure of the $Y$ system (e.g. $D \pi \pi^0$, $m_{\pi \pi^0} < 1.5$ GeV/$c^2$). For each mode, the most relevant parameter is the apriori-purity of the mode: the ratio $S/\sqrt{S+B}$, where $S$ and $B$ are the signal and combinatorial background respectively, as estimated from a $m_{ES}$ fit on data (Sec.3.5.2);

- determine a mode by mode combinatorial background rejection, in order to account for different background levels depending on the number of charged tracks and, above all, on the number of $\pi^0$s in the reconstructed mode (Sec.3.5.3);

- rank the submodes according to their purity and yields and study the significance as a function of the number of used modes in order to maximize the statistical significance of the sample (Sec. 3.5.4);

- group the submodes with similar purity;

- resolve the multiple candidates (Sec. 3.5.4).

The starting point of the Semi-exclusive selection is the $D^0, D^+, D^*, D^{\ast 0}$ meson reconstruction, described in Sec. 3.4.3. Charged pions and kaons, $\pi^0$, and $K^0_S$ candidates are then combined to the $D$ meson to form the $B$ candidate.

EVENT RECONSTRUCTION
3.5.1 Definition of $\Delta E$ and $m_{ES}$

Two main variables are used to select $B$ candidates, to extract the yields and to define a sideband region to study the background: $\Delta E$ and $m_{ES}$.

The energy difference $\Delta E$ is defined as:

$$\Delta E = E_B^* - \sqrt{s}/2,$$

where $E_B^*$ is the energy of the $B$ candidate in the $\Upsilon(4S)$ rest frame (CM) and $\sqrt{s}$ is the total energy of the $e^+e^-$ system in the CM rest frame. The resolution of this variable is affected by the detector momentum resolution and by the particle identification since a wrong mass assignment results in a shift in $\Delta E$. Due to the energy conservation, signal events are Gaussian distributed in $\Delta E$ around zero. Continuum and part of the $b\bar{b}$ background have a $\Delta E$ distribution that can be modeled with a polynomial distribution. Instead, some other $b\bar{b}$ background, due to misidentification, gives shifted Gaussian peaks. The resolution of this variable depends essentially on the reconstructed $B$ mode and $\pi^0$ multiplicity and it can varies from 20 to 40 MeV.

The beam energy-substituted mass $m_{ES}$ is defined as

$$m_{ES} = \sqrt{(\sqrt{s}/2)^2 - p_B^*^2},$$

where $p_B^*$ is the $B$ candidate momentum in the CM rest frame. It is clear that, since $|p_B^*| \ll \sqrt{s}/2$, the experimental resolution on $m_{ES}$ is dominated by beam energy fluctuations. To an excellent approximation, the shapes of the $m_{ES}$ distributions for $B$ meson reconstructed in a final states with charged tracks only are Gaussian and practically identical. Otherwise the presence of neutrals in the final states, in case their showers are not fully contained in the calorimeter, can introduce tails.

It is important to notice that, since the sources of experimental smearing are uncorrelated (beams energy for $m_{ES}$ and detector momentum resolution for $\Delta E$), $m_{ES}$ and $\Delta E$ also are basically uncorrelated.

The background shape in $m_{ES}$ is parameterized using the ARGUS function [65]:

$$\frac{dN}{dm_{ES}} = N \cdot m_{ES} \cdot \sqrt{1 - x^2} \cdot \exp \left(-\xi \cdot (1 - x^2)\right)$$

where $x = m_{ES}/m_{max}$ and $\xi$ is a free parameter determined from the fit. The $m_{max}$, that represents the endpoint of the ARGUS distribution, is fixed in the fit to $m_{ES}$, since it depends only on the beam energy. The ARGUS function provides a good parameterization of the continuum ($c\bar{c}$ and $ud$) and $b\bar{b}$ background events, as shown in Fig.3-9.

The signal component is fitted using a Crystal Ball function [66]:

- if $m_{ES} > m - \sigma \cdot a$

$$\frac{dN}{dm_{ES}} = N \cdot \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \cdot \exp \left(-\frac{1}{2} \cdot \frac{(m_{ES} - m)^2}{\sigma^2}\right)$$

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\[ \frac{dN}{dm_{ES}} = N \cdot \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \cdot \left( \frac{m}{\alpha} \right)^n \exp \left( -\frac{1}{2} \cdot \frac{m^2}{\sigma^2} \right) \frac{1}{\left( \frac{m_{ES} - m}{\sigma} + \frac{m}{\alpha} - \alpha \right)^n} \]  \quad (3.8)

The radiative tail of this function can takes into account cases where the energy of the neutral candidates is not fully deposited in the EMC crystals. The left tail of the distribution depends on the reconstructed \( B \) mode and in particular on the number of \( \pi^0 \). Fig.3-10 shows the fitted shape on the Monte Carlo for modes with no \( \pi^0 \), one \( \pi^0 \) an two \( \pi^0 \).

The maximum total number of floating parameters in the \( m_{ES} \) fits is 7. Two of them are for the ARGUS shape, while the remaining five parameters are for the Crystal Ball one.

In the following the number of signal and background events (indicated as \( S \) and \( B \) in the plots) are estimated as the area of the Crystal Ball and the ARGUS functions integrated for \( m_{ES} > 5.27 \text{ GeV} \).

3.5.2 Study of the Y system

The choice of the submodes is crucial in the reconstruction method. The identification of the clean modes allows to set up the most efficient and pure selection among the multiple candidates in different modes. A detailed study of the Y system, looking for resonances in the signal and background shape is performed.
We consider, for example, the mode $B \rightarrow D^* \pi \pi^0$. The invariant mass of the $Y = \pi \pi^0$ system is shown in Fig.3-11. There is a large contribution below 1.5 GeV/$c^2$ due to the $\rho$ resonance, but there is also a small amount of signal at $\sim 2.4-2.6$ GeV/$c^2$, but not clear enough for a specific selection. Therefore two sub-modes are defined depending on whether $m_{\pi \pi^0}$ is smaller than 1.5 GeV/$c^2$ or greater than 1.5 GeV/$c^2$, without requiring the sub-mode belonging to a precise resonance structure. In this way the clean $B \rightarrow D^* \pi \pi^0$ sub-mode ($m_{\pi \pi^0} > 1.5$ GeV/$c^2$) has been separated from the low purity ones ($m_{\pi \pi^0} < 1.5$ GeV/$c^2$).

Finally, the total number of the reconstructed $B$ decay modes is 52 (53 for the $D^+$ seed). The total number of the decay modes is 1097. A summary is shown in Tab. 3-9.

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### 3.5.3 $\Delta E$ selection

Once all possible reconstruction modes are identified, a window in $\Delta E$ is applied in order to pick up among several candidates in a given submode.

The $\Delta E$ resolutions are determined from the $\Delta E$ distributions before the request of the best candidates and they depend essentially on the number of charged tracks and, above all, on the number of $\pi^0$'s in Y system (since the reconstructed $D$ is mass constrained). For the modes without $\pi^0$'s a fit with a linear background and a Gaussian is performed and 2 $\sigma$ symmetric windows are taken. In the case of modes with at least a $\pi^0$, the situation is worse. First of all there are too many candidates per event. Requiring that only the 10 candidates with the smallest $|\Delta E|$ are taken, can create a bias in the $\Delta E$ distribution. Due to studies of the decay modes, the chosen windows are: $|\Delta E| < 45$ MeV for candidates without $\pi^0$'s and $K_S^0$'s,
$|\Delta E| < 50\text{ MeV}$ for candidates with up to one $\pi^0$ and two $K_S^0$ 's and $-90 < \Delta E < 60\text{ MeV}$ for all the others.

### 3.5.4 Multiple candidates and definition of purity

Two kinds of multiple candidates are possible: multiple candidates can be reconstructed in the same submode and many submodes per event are also possible.

If there are multiple candidates in the same submode only the one with lowest $\Delta E$ is chosen and one candidate per submode is selected.

The selection of the best $B$ decay among different submodes cannot use $\Delta E$ because the modes with higher combinatoric background would be privileged with respect to the clean ones. An unbiased criterion for choosing a signal event is based on a $a$-priori probability. The $a$-priori probability here is given by the purity of the mode, determined by fitting the $m_{ES}$ distribution and determining the signal and background contributions in the signal region ($m_{ES} > 5.27\text{ GeV}$ ). The selection of the best $B$ in the event is based on the choice of the reconstructed mode with the highest purity.

The decay modes are ranked according to their purity and are added to the sample of reconstructed $B$ 's one at a time. At each addition of a mode the yield increases and the purity mostly decreases. This method is very useful once the composition of the modes has to be optimized for the analysis of the recoil. The significance $S/\sqrt{S+B}$ is computed as a function of the number of added modes and the best composition is chosen. An example for the $B^0 \rightarrow D^+ Y$ case is shown in Fig.3-12.

The final yields depend on the cut on the purity. The yields for different levels of purity on the $B_{reco}$ sample are shown in Tab.3-10.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$a$-priori pur. $&gt; 80%$</th>
<th>$a$-priori pur. $&gt; 50%$</th>
<th>$a$-priori pur. $&gt; 10%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow D^0 X$</td>
<td>$75524 \pm 322$</td>
<td>$213774 \pm 684$</td>
<td>$376055 \pm 1316$</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^+ X$</td>
<td>$43726 \pm 258$</td>
<td>$101594 \pm 518$</td>
<td>$220528 \pm 948$</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^*0 X$</td>
<td>$73470 \pm 337$</td>
<td>$174866 \pm 665$</td>
<td>$297632 \pm 1161$</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*+ X$</td>
<td>$81646 \pm 335$</td>
<td>$198685 \pm 676$</td>
<td>$219462 \pm 768$</td>
</tr>
<tr>
<td>Total $B^+$</td>
<td>$148994 \pm 477$</td>
<td>$388640 \pm 1010$</td>
<td>$673712 \pm 1752$</td>
</tr>
<tr>
<td>Total $B^0$</td>
<td>$125372 \pm 414$</td>
<td>$300279 \pm 871$</td>
<td>$439990 \pm 1232$</td>
</tr>
<tr>
<td>Total</td>
<td>$274366 \pm 631$</td>
<td>$688919 \pm 1308$</td>
<td>$1113702 \pm 2116$</td>
</tr>
</tbody>
</table>

**Table 3-10.** Yields from Semi-exclusive reconstruction for different levels of purity for 316 $fb^{-1}$ of data.

The study of the recoiling $B$ can improve the purity of the sample, since the application of selection criteria on the recoil, for instance the request of an hard lepton, removes most of the non $b\bar{b}$ events without changing the $m_{ES}$ shape above the signal threshold ($5.27\text{ GeV}$ ) and allows to use most of the Semi-exclusive modes.

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Figure 3-12. Dependence of the quality factor $S/\sqrt{S+B}$ as a function of the yield when adding modes for the $B^0 \rightarrow D^+ Y$ case. Statistic corresponds to 80 fb$^{-1}$. 
4

Reconstruction of signal side:

\[ B \rightarrow X\ell\nu \] selection

4.1 Introduction

The \( B \) meson recoiling against the \( B_{\text{reco}} \) is reconstructed exclusively in order to select a charmless semileptonic decay \( B \rightarrow X_u\ell\nu \). Here the hadronic system \( X_u \) represents any of the following charmless mesons: \( \pi^0, \pi^\pm, \eta \) and \( \eta' \). This section describes the reconstruction of the \( X_u \) mesons used in the exclusive reconstruction of the \( B_{\text{recoil}} \).

The study the \( B \rightarrow X_u\ell\nu \) decays proceeds as follows:

- A sample with at least one fully reconstructed \( B \) is selected (as discussed in Chapter 4).
- Charged tracks and neutral clusters that are not associated to the \( B_{\text{reco}} \) are identified as described in Sec.3.1 and 3.3.
- The \( X_u \) system for the channels under study is reconstructed and selected as discussed in Sec.4.2.
- A sample of semileptonic \( B \) decay on the recoil is identified by requiring the presence of a charged lepton. Leptons are identified using algorithms described in Sec.3.2.1 and 3.2.2.
- The different \( B \rightarrow X_u\ell\nu \) modes are selected with specific criteria to enrich the sample of signal events. These criteria are described in Sec.4.3.2.

4.2 Exclusive reconstruction of the \( X_u \) system

In this section the reconstruction of the \( X_u \) system is discussed. Selection criteria described here refer to the pre-selection and have been optimized for the final selection.

All the distributions for signal events have been obtained using the high statistic pure resonant Monte Carlo sample. The plots showing signal and background components have been obtained using \textit{Generic} Monte Carlo samples.

In the following plots four categories of events are displayed:

- \textbf{signal}: events belonging to the particular charmless semileptonic channel under study;
• **other** $b \rightarrow u \ell \nu$ : events from other semileptonic charmless decays;

• $b \rightarrow c \ell \nu$ : events from semileptonic decays containing charmed mesons;

• **other** : background events from other sources, such as misidentified leptons, secondary $\tau$ and charm decays.

A summary of the properties of the reconstructed resonances and their relative selection criteria are shown in Tab.4-1.

### 4.2.1 $\pi^0$ reconstruction

Composite $\pi^0$ ’s are defined as pairs of photons.

The $\pi^0$ candidates are reconstructed as pairs of neutral candidates measured in the EMC and are selected with two different selection criteria. The loose criterion uses *loose* photons (see Sec.3.3) and requires the reconstructed mass to be within the range $0.110 < m_{\pi^0} < 0.160 \text{ GeV}/c^2$. The tight criterion uses *tight* photons and requires the reconstructed mass to be within the range $0.115 < m_{\pi^0} < 0.150 \text{ GeV}/c^2$.

$\pi^0$ ’s candidate used to reconstruct the $B^0 \rightarrow \pi^- \ell^+ \nu$ decay modes are selected using the loose criteria but a tighter cuts on the photon energy in the laboratory $E_\gamma > 80 \text{ MeV}$ is required to reduce the combinatorial background.

Fig.4-1 shows the resulting $\pi^0$ mass distribution on *Signal* Monte Carlo after all selection criteria for $B^0 \rightarrow \pi^- \ell^+ \nu$ mode have been applied (see Sec.4.3.2). The reconstructed efficiency of $\pi^0$ ’s is $49.8 \pm 1.7\%$.

![Reconstructed mass for pi0](image)

**Figure 4-1.** Reconstructed $\pi^0$ invariant mass in signal MC for $B^0 \rightarrow \pi^- \ell^+ \nu$ decay mode. All selection criteria have been applied.

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4.2.2 $\eta$ reconstruction

We reconstruct $\eta$ candidates in the following modes:

- $\eta \rightarrow \gamma \gamma$ ($BF = 39.4\%$): $\gamma$’s are reconstructed as pairs of photons with a cut on the photon momentum in the laboratory at $p_\gamma > 30$ MeV/c.

- $\eta \rightarrow \pi^+ \pi^- \pi^0$ ($BF = 22.6\%$): $\pi$’s are reconstructed by using pairs of charged tracks with opposite charge and one loose $\pi^0$.

- $\eta \rightarrow \pi^0 \pi^0 \pi^0$ ($BF = 32.5\%$): $\pi^0$’s are combinations of three $\pi^0$’s. In order to reduce the combinatorial background we use two loose $\pi^0$’s and a tight one.

We require for $\eta$ candidates a reconstructed mass within the range $0.45 < m_\eta < 0.65$ GeV/$c^2$ (same for all modes). In Fig. 4-2 the reconstructed $\eta$ invariant mass for the three different decay modes is shown. The plots are obtained using Signal Monte Carlo and the semileptonic selection described in Sec. 4.3.1. We performed a fit to the mass distributions using the sum of a Gaussian for the signal component and a first order polynomial for the combinatoric background, in order to evaluate the mass resolutions for each mode. The results of this fits has been not used to extract final results.

Taking properly into account the $\eta$ decay branching fractions and the reconstruction efficiency for the different $\eta$ decay modes, the total weighted efficiency of the $\eta$ reconstruction is $36.2 \pm 1.2\%$. In Fig.4-3 the relative contribution of each reconstructed mode to the sample of $\eta$ candidates is shown.

4.2.3 $\eta'$ reconstruction

The reconstruction of the $\eta'$ candidates is based on the following modes:

- $\eta' \rightarrow \rho^0 \gamma$ ($BF = 29.5\%$): $\rho$’s are reconstructed as a combination of one $\rho^0$ and one loose photon. The $\rho$ candidates are reconstructed using pairs of charged tracks with opposite charge and no mass cut is applied at this level. The energy of the photon in the recoiling B rest frame is used to reject background. We apply a cut at $p_\gamma^* > 0.35$ GeV/c. In particular, this cut is very effective to remove cross-feed from $B^+ \rightarrow \rho^0 \ell^+ \nu$ and $b \rightarrow c \ell \nu$ decays. The photon energy is shown in Fig.4-4.

- $\eta' \rightarrow \eta \pi^+ \pi^-$ ($BF = 44.3\%$): $\eta$’s are reconstructed as combinations of pairs of tracks with opposite charge and one $\eta$. The $\eta$ candidates are reconstructed in three modes ($\eta \rightarrow \gamma \gamma$, $\eta \rightarrow \pi^+ \pi^- \pi^0$, $\eta \rightarrow \pi^0 \pi^0 \pi^0$), using the same selection described in Sec.4.2.2.

$\eta'$’s are required to have a reconstructed mass within the range $0.86 < m_{\eta'} < 1.06$ GeV/$c^2$ (same for all modes).

In Fig.4-5 the reconstructed $\eta'$ invariant mass for the four different decay modes is shown. The plots are obtained using Signal Monte Carlo and the semileptonic selection is applied. As for the $\eta$ candidates, we
performed a fit to the $\eta'$ mass distributions using the sum of a Gaussian for the signal component and a first order polynomial for the combinatoric background, in order to evaluate the mass resolutions for each mode. The results of this fits has been not used to extract final results.

Taking properly into account the $\eta'$ and $\eta$ decay branching fractions and the reconstruction efficiency for the different decay modes, the total weighted efficiency of the $\eta'$ reconstruction is 20.2 ± 0.6%.

In Fig.4-6 the relative contribution of each reconstructed mode to the sample of $\eta'$ candidates is shown.

### 4.3 Event Selection

The event selection criteria are optimized to obtain a sample of semileptonic $B$ decays among which several exclusive charmless semileptonic events can be selected.

The analysis makes therefore use of a two-step selection. We first select semileptonic events (“semileptonic selection”) and then we refine the sample applying mode specific criteria (“signal selection”). These cuts
4.3 Event Selection

Figure 4-3. Relative contribution of each reconstructed mode to the sample. 1 bin: $\eta \rightarrow \gamma \gamma$, 2 bin: $\eta \rightarrow \pi^+ \pi^- \pi^0$, 3 bin: $\eta \rightarrow \pi^0 \pi^0 \pi^0$.

<table>
<thead>
<tr>
<th>$X_{\alpha}$</th>
<th>$m_{X_{\alpha}}$ (MeV)</th>
<th>decay channel</th>
<th>$BR(%)$</th>
<th>daughter, cuts</th>
<th>mass window (GeV)</th>
<th>$\epsilon(%)$</th>
<th>$\epsilon \times BR(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>$m = 139.57$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\pi$ Charged Tracks</td>
<td></td>
<td></td>
<td>$68.7 \pm 2.2$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$m = 134.97$</td>
<td>$\gamma \gamma$</td>
<td>$98.8 \pm 0.03$</td>
<td>$\gamma$ (EMC neutrals) $p_T &gt; 80$ MeV</td>
<td>$0.11 &lt; m_{\pi^0} &lt; 0.16$</td>
<td>$49.8 \pm 1.7$</td>
<td>$49.2 \pm 1.7$</td>
</tr>
<tr>
<td></td>
<td>$m = 771$</td>
<td>$\pi^+ \pi^0$</td>
<td>$100$</td>
<td>$\pi^+ (\pi^0)$ see above</td>
<td>$p_T &gt; 300$ MeV/c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>$m = 771$</td>
<td>$\pi^+ \pi^-$</td>
<td>$100$</td>
<td>$\pi^+ (\pi^-)$ see above, higher energy $\pi$ with $p_T &gt; 350$ MeV/c</td>
<td>$0.45 &lt; m_{\pi^+} &lt; 0.65$</td>
<td>$40.0 \pm 2.7$</td>
<td>$36.0 \pm 1.8$</td>
</tr>
<tr>
<td></td>
<td>$\rho^0$</td>
<td>$\pi^0 \pi^0$</td>
<td>$89.1$</td>
<td>(see above)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>$m = 782.6$</td>
<td>$\pi^+ \pi^0 \pi^0$</td>
<td>$0.68 &lt; m_{\omega} &lt; 0.88$</td>
<td>$28.7 \pm 0.9$</td>
<td>$\omega \pi^+ \pi^-$</td>
<td>$36.2 \pm 1.2$</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>$m = 547.8$</td>
<td>$\gamma \gamma$</td>
<td>$39.4$</td>
<td>$\gamma$ (EMC neutrals) $p_T &gt; 30$ MeV</td>
<td>$0.45 &lt; m_{\eta} &lt; 0.65$</td>
<td>$51.8 \pm 2.1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pi^+ \pi^- \pi^0$</td>
<td>$22.6$</td>
<td>$\pi^+ (\pi^-)$ see above, higher energy $\pi$ with $p_T &gt; 350$ MeV/c</td>
<td>$0.45 &lt; m_{\eta} &lt; 0.65$</td>
<td>$40.0 \pm 2.7$</td>
<td>$21.8 \pm 2.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pi^0 \pi^+ \pi^-$</td>
<td>$32.5$</td>
<td>$\pi^0 \pi^+ (\pi^-)$ see above, higher energy $\pi$ with $p_T &gt; 350$ MeV/c</td>
<td>$0.45 &lt; m_{\eta} &lt; 0.65$</td>
<td>$33.4 \pm 2.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta'$</td>
<td>$m = 957.8$</td>
<td>$\rho^0 \gamma$</td>
<td>$29.5$</td>
<td>$\rho^0$ from $\pi^\pm$ Charged Track $\gamma$ (EMC neutrals) $p_T &gt; 0.35$ GeV</td>
<td>$0.66 &lt; m_{\eta'} &lt; 0.96$</td>
<td>$42.1 \pm 2.4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\eta' \pi^+ \pi^-$</td>
<td>$44.3$</td>
<td>$\pi^+ \pi^-$ see above, higher energy $\pi$ with $p_T &gt; 350$ MeV/c</td>
<td>$0.66 &lt; m_{\eta'} &lt; 0.96$</td>
<td>$33.4 \pm 2.1$</td>
<td>$21.8 \pm 2.5$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4-1. Summary of all reconstructed resonances

RECONSTRUCTION OF SIGNAL SIDE: $B \rightarrow X \ell \nu$ SELECTION
are aimed at rejecting two kinds of backgrounds: the $b \to c$ jets and charmless semileptonic $B$ decays other than the signal ones. For cuts on the $X_{q}$ mass distributions the separation power of each variable by trying to retain as much signal as possible has been exploited, while the selection criteria on the other variables have been optimized by minimizing the expected statistical error of the measurement.

4.3.1 Semileptonic Selection

This selection aimed at selecting events with an energetic prompt lepton in the recoil of a fully reconstructed $B$:

- reconstructed $B$ modes and cut on the purity per seed. The presence of a fully reconstructed $B$, neutral for $B^{0} \to \pi^{-} \ell^{+}\nu$ and charged for $B^{+} \to \pi^{0} \ell^{+}\nu$, $B^{+} \to \eta\ell^{+}\nu$ and $B^{+} \to \eta'\ell^{+}\nu$ is required. The semi-exclusive reconstruction allows for a selection of the purity of the sample. The selection criteria on the purity of the fully reconstructed $B$ candidates for this analysis are summarized in Tab. 4-2. The optimization of criteria on the purity for the $B_{\text{rec}}$ modes has been performed studying the impact of the purity selection on the statistical error on the basis of data.

- Lepton momentum in the $B_{\text{rec}}$ rest frame.

Semileptonic $B$ decays are identified by the presence of a high momentum electron or muon. To reduce the backgrounds from secondary charm or $\tau^{\pm}$ decays and from fake leptons, a minimum lepton momentum in the $B$ rest frame is required. The boost to the rest frame of recoiling $B$ is possible since the momenta of the $Y(4S)$ and of the reconstructed $B$ are known. Since the hadronic tag technique assures that the level of $b \to c\ell\nu$ background is low, very loose selection criteria can be applied. For

![Energy of the photon in the recoiling B rest frame for reconstructed $\eta' \to \rho\gamma$ for Signal Monte Carlo (left) and Generic Monte Carlo (right).](image)

**Figure 4-4.** Energy of the photon in the recoiling $B$ rest frame for reconstructed $\eta' \to \rho\gamma$ for Signal Monte Carlo (left) and Generic Monte Carlo (right).
4.3 Event Selection

Figure 4-5. Reconstructed $\eta'$ invariant mass for $\eta' \rightarrow \rho \gamma$ (top left) and $\eta' \rightarrow \eta \pi^+ \pi^-$ with $\eta \rightarrow \gamma \gamma$ (top right), $\eta \rightarrow \pi^+ \pi^- \pi^0$ (bottom left) and $\eta \rightarrow \pi^0 \pi^0 \pi^0$ (bottom right) on Signal Monte Carlo. All selection criteria have been applied.

all modes we request the presence of at least one lepton in the recoil with $p_T > 0.5 \text{ GeV}/c$ if the

Figure 4-6. Relative contribution of each reconstructed mode to the sample. 1 bin: $\eta' \rightarrow \rho \gamma$, 2 bin: $\eta' \rightarrow \eta \pi^+ \pi^-$ ($\eta \rightarrow \gamma \gamma$), 3 bin: $\eta' \rightarrow \eta \pi^+ \pi^-$ ($\eta \rightarrow \pi^+ \pi^- \pi^0$), 4 bin: $\eta' \rightarrow \eta \pi^+ \pi^-$ ($\eta \rightarrow \pi^0 \pi^0 \pi^0$).

RECONSTRUCTION OF SIGNAL SIDE: $B \rightarrow X \ell \nu$ SELECTION
Table 4-2. Optimum choice of the purity of the sample.

<table>
<thead>
<tr>
<th>seed mode</th>
<th>purity cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D^* Y$</td>
<td>24%</td>
</tr>
<tr>
<td>$B^0 \to D^+ Y$</td>
<td>9%</td>
</tr>
<tr>
<td>$B^{\pm} \to D^{*0} Y$</td>
<td>8%</td>
</tr>
<tr>
<td>$B^{\pm} \to D^0 Y$</td>
<td>7%</td>
</tr>
</tbody>
</table>

lepton is identified as an electron and $p_t^e > 0.8 \text{ GeV}/c$ if the lepton is identified as a muon. This cut keeps basically the full phase space. Thus, the dependence on the form factors and the different theoretical models in the extraction of the branching fractions is mostly eliminated.

- Lepton Charge and $B$ Flavor Correlation.

In semileptonic decays the charge of the primary lepton is correlated with the charge of the flavour of $b$ quark. This means that $Q_{b(\text{reco})} Q_t < 0$ for primary leptons and $Q_{b(\text{reco})} Q_t > 0$ for secondary leptons (here $Q_{b(\text{reco})}$ refers to the $b$-quark charge and $Q_t$ to the lepton charge in the semileptonic decay), which are due to cascade decays of the charm particle in $b \to c\ell\nu$ transitions. Unfortunately the charge of $B_{\text{reco}}$ is known exactly only in the case of charged $B$ decays. Therefore, for reconstructed charged $B$ mesons we require $Q_{b(\text{reco})} Q_t < 0$, instead, in the case the reconstructed $B$ meson is neutral, the effect of the $B^0 - \bar{B}^0$ mixing needs to be taken into account. If the sample was made only of direct and cascade leptons from $B$ decays, the right sign ($rs$) and the wrong sign ($ws$) events would be related to the direct ($B$) and cascade ($\bar{B}$) decays by:

$$N_{rs} = 1 - \chi_d N_B + \chi_d N_D$$  \hspace{1cm} (4.1) $$N_{ws} = \chi_d N_B + 1 - \chi_d N_D$$  \hspace{1cm} (4.2)

where $\chi_d = 0.186 \pm 0.004$ [59] is the mixing parameter. For reconstructed neutral $B$ mesons, both events, ($rs$) and ($ws$), are accepted and the sample is subsequently correct for the effects of the $B^0 - \bar{B}^0$ mixing:

$$N_B = \frac{1 - \chi_d}{1 - 2\chi_d} N_{rs} - \frac{\chi_d}{1 - 2\chi_d} N_{ws}.$$  \hspace{1cm} (4.3)

### 4.3.2 Signal Selection

On top of the Semileptonic Selection these cuts are devised to select our specific signals:

- Number of leptons

  In $b \to c\ell\nu$ transitions a second lepton is very frequent due to cascade decays of the charm particles. In contrast, in $b \to u\ell\nu$ decays, secondary leptons are very rare. Thus for the $b \to u\ell\nu$ signal sample one and only one lepton with $p_t^e > 0.5 \text{ GeV}/c$ or $p_t^\mu > 0.8 \text{ GeV}/c$ is requested in the event.
4.3 Event Selection

- **Total Charge of the Event.**
  Since one $B$ meson is fully reconstructed, charge conservation $Q_{\text{tot}} = Q_{B_{\text{reco}}} + Q_{B_{\text{reco}}} = 0$ is imposed for all modes. This cut rejects preferentially $b \to c \ell \nu$ events, since their higher charge multiplicity leads to higher loss of charged tracks, but also events with a missing charged particle and events with an additional charged particle due to $\gamma \to e^+ e^-$ conversions or tracking errors.

- **Charged Track Multiplicity.**
  We request that no other tracks are allowed to be present in the recoil except for the ones used to reconstruct the resonance and the lepton. From Monte Carlo estimation we get that allowing one or more undetected charged tracks is not effective in increasing the number of signal events, while a large fraction of background events, mainly $b \to c \ell \nu$, is maintained.

- **Missing Mass Squared.**
  We define the four momentum of the missing particle on the recoil as:
  \[
  P_{\text{miss}} = P_{\text{T}(4S)} - P_{B_{\text{reco}}} - P_{X_u} - P_{\ell}
  \] (4.4)
  where $P_{\text{T}(4S)}$ is the sum of the four-momenta of the colliding beams, $P_{B_{\text{reco}}}$ is the measured four-momenta of the $B_{\text{reco}}$, $P_{X_u}$ is the measured four-momenta of the $\pi, \pi^0, \eta$ or $\eta'$ candidate and the $P_{\ell}$ is the measured four-momenta of the lepton. The missing mass squared is defined as $m_{\text{miss}}^2 = P_{\text{miss}}^2$ and it’s an important estimator of the quality of the reconstruction of the recoil system.

  In semileptonic $B$ decays the only undetected particle should be the neutrino. Thus a cut on the missing mass is a very powerful tool to reject events in which one or more particle is undetected or very poorly measured. The $m_{\text{miss}}^2$ distribution is much broader and extends to higher values for $b \to c \ell \nu$ decays, thus a cut results in a valuable suppression of this background. This suppression is due to higher multiplicities and/or the presence of an additional neutrino or $K_L$ in charm decays.

  The $m_{\text{miss}}^2$ distribution for the different contributions (signal, background $b \to u \ell \nu$, $b \to c \ell \nu$ and other) is displayed in Fig.4-7 and 4-8. Plots show the $m_{\text{miss}}^2$ resolution for all modes and the discriminant power of this variable. The resolution on $m_{\text{miss}}^2$ is particularly good thanks to the fact that the rest of the event is completely reconstructed. The cut on $m_{\text{miss}}^2$ has been optimized for each mode by maximizing the statistical significance. The different optimal cuts are detailed in Tab.4-3.

- **Hadronic mass window**
  The selection criteria on the reconstructed masses $m_{X_u}$ has been optimized for each mode.
  For $B^+ \to \pi^0 \ell^+ \nu$ mode we chose the $\pi^0$ mass window the maximize the statistical significance on Generic Monte Carlo.
  For $B^+ \to \eta \ell^+ \nu$ and $B^+ \to \eta' \ell^+ \nu$ modes the selection criteria on the $\eta$ and $\eta'$ reconstructed masses has been optimized using a different procedure. Since the mass resolution is different for the different decay modes we performed a fit to each reconstructed mass using a Gaussian distribution for the signal component and a first order polynomial for the background in order to determine the resolution. Then we selected a window of $2.5\sigma$ around the nominal value [59]. The fitted mass distributions for $\eta$ and $\eta'$ are shown in Fig.4-2 and Fig. 4-5.
  The selected mass windows for all resonances are reported in Tab.4-3.

**RECONSTRUCTION OF SIGNAL SIDE: $B \to X \ell \nu$ SELECTION**
Multiple Candidates

For each decay mode, when multiple candidates of a given resonance are reconstructed, we first keep all reconstructed resonances in the event passing all selection criteria and then the one with the event based $m_{miss}^2$ closest to zero is chosen.

As concern $B^0 \rightarrow \pi^- \ell^+ \nu$, since we require only two charged tracks in the recoil side and moreover we select the charged pion among all tracks in the recoil not overlapping with the lepton, multiple $B^0 \rightarrow \pi^- \ell^+ \nu$ candidates are not present.

Special cuts

In order to further reduce the source of backgrounds, which still affect the sample of signal events after the cuts described above, we apply a set of particular cuts which exploit the particular topology of the single decay modes.

- $B^+ \rightarrow \pi^0 \ell^+ \nu$ mode
In order to further reject combinatoric background we apply a cut on the momentum of the most energetic photon, used to reconstruct the $\pi^0$, in the $B_{recoil}$ rest frame. Based on Monte Carlo estimation, we require at least a photon with momentum $p_\gamma > 300\,\text{MeV}/c$, which keeps above the 98% of signal photons. In Fig. 4-9 the reconstructed energy distribution for the most energetic photon in Signal and Generic Monte Carlo are shown.

- $B^0 \rightarrow \pi^- \ell^+\nu$ mode

For this decay mode, the $J/\psi \rightarrow \ell^+\ell^-$ events introduce a background due to the mis-identification of a lepton as a $\pi$. In order to remove this background, the lepton mass hypothesis is applied to the charged pion and the invariant mass $m_{\ell\pi}$ of the lepton-$\pi$ pair is requested to be outside the range $3.08 < m_{\ell\pi} < 3.12\,\text{GeV}/c^2$.

Moreover, to reduce background coming from $B^0 \rightarrow \rho\ell\nu$ a cut on the neutral energy $E_{\text{neutral}} < 450\,\text{MeV}$ is applied.

- $B^+ \rightarrow \eta\ell^+\nu$ mode

**Reconstruction of Signal Side: $B \rightarrow X\ell\nu$ Selection**
Reconstruction of signal side: $B \rightarrow X\ell\nu$ selection

Figure 4-9. Reconstructed energy for the most energetic photon used to reconstruct a $\pi^0$ calculated in the recoiling $B$ meson rest frame. Signal MC after all cuts (left) and Generic MC after lepton cuts (right).

The main source of background for $B^+ \rightarrow \eta\ell^+\nu$ is due to $B^+ \rightarrow \pi^0\ell^+\nu$ events. We apply a cut on the missing mass squared of the event, calculated assuming the $\pi^0$ mass hypothesis, $|m_{\text{miss}}^2 (\pi^0)| > 1.5 \text{GeV}/c^2$ to reject this background. Fig.4-10 shows the $m_{\text{miss}}^2$ distribution for signal and $|V_{ub}|$ cross-feed events.

- $B^+ \rightarrow \eta'\ell^+\nu$ mode

for $B^+ \rightarrow \eta'\ell^+\nu$, in order to reject $b \rightarrow c\ell\nu$ background, we tighten the cut on the mass of the daughters of the $\eta'$ meson. We apply a cut at $|m_{\rho^0} - m_{\rho^0}^{PDG}| < 180 \text{MeV}$ on the $\rho^0$ reconstructed mass. For $\eta'$s originating from a $\eta'$ we determine different mass windows around the nominal value, depending on the $\eta$ decay mode. The resulting mass windows are reported in Tab. 4-3.

4.4 Data/Monte Carlo Validation

A good description of the relevant variables by the Monte Carlo simulation is important for this analysis. Therefore, a comparison between data and Monte Carlo is performed to check how well the simulation models the data. We used the whole statistic available for data and Generic Monte Carlo.

Fig. 4-11, 4-12, 4-13, 4-14 and 4-15 show the data - Monte Carlo comparisons of various variables used in the analysis. All the distributions are shown after only lepton cut and after all selection criteria have been applied, except the one on the plotted variable. The combinatoric background below 5.27 GeV in $m_{ES}$ has been subtracted in all spectra, performing a fit of the $m_{ES}$ distribution with the sum of a Crystal Ball function and an ARGUS function (see Sec.3.5.1). The histograms for data and Monte Carlo are normalized to equal area.

Alessia D’Orazio
<table>
<thead>
<tr>
<th>Selection</th>
<th>common cuts for all modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton momentum</td>
<td>$p_{el}^* &gt; 0.5 \text{ GeV}/c \cdot p_{\mu}^* &gt; 0.8 \text{ GeV}/c$</td>
</tr>
<tr>
<td>Number of leptons</td>
<td>$N_{\text{lepton}} = 1$</td>
</tr>
<tr>
<td>Charge conservation</td>
<td>$Q_{\text{tot}} = 0$</td>
</tr>
<tr>
<td>Number of tracks</td>
<td>no additional charged tracks</td>
</tr>
<tr>
<td>Charge correlation</td>
<td>$Q_{b(\text{reco})} Q_{\ell} &lt; 0$ (and mixing correction)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B^0 \rightarrow \pi^- \ell^+ \nu$</th>
<th>$B^+ \rightarrow \pi^0 \ell^+ \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missing mass squared ( \pi^0 ) mass</td>
<td>$</td>
</tr>
<tr>
<td>Neutral energy $J/\psi \rightarrow \ell^+ \ell^-$</td>
<td>$E_{\text{neutral}} &lt; 0.45 \text{ GeV}$</td>
</tr>
<tr>
<td>background rejection</td>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B^+ \rightarrow \eta \ell^+ \nu$</th>
<th>$B^+ \rightarrow \eta' \ell^+ \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missing mass squared ( \pi^0 ) mass</td>
<td>$</td>
</tr>
<tr>
<td>$\eta$ mass</td>
<td>$115 &lt; m_{\gamma\gamma} &lt; 150 \text{ MeV}/c^2$ ($\eta \rightarrow \pi^0 \pi^0 \pi^0$)</td>
</tr>
<tr>
<td>($\eta \rightarrow \gamma\gamma$)</td>
<td>$505 &lt; m_\eta &lt; 585 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>($\eta \rightarrow \pi^+ \pi^- \pi^0$)</td>
<td>$530 &lt; m_\eta &lt; 560 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>($\eta \rightarrow \pi^0 \pi^0 \pi^0$)</td>
<td>$510 &lt; m_\eta &lt; 580 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>$\eta'$ mass</td>
<td></td>
</tr>
<tr>
<td>($\eta' \rightarrow \rho^0\gamma$)</td>
<td></td>
</tr>
<tr>
<td>($\eta' \rightarrow \eta \pi^+ \pi^-$, $\eta \rightarrow \gamma\gamma$)</td>
<td></td>
</tr>
<tr>
<td>($\eta' \rightarrow \eta \pi^+ \pi^-$, $\eta \rightarrow \pi^+ \pi^- \pi^0$)</td>
<td></td>
</tr>
<tr>
<td>($\eta' \rightarrow \eta \pi^+ \pi^-$, $\eta \rightarrow \pi^0 \pi^0 \pi^0$)</td>
<td></td>
</tr>
<tr>
<td>$\rho^0$ mass</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ momentum</td>
<td></td>
</tr>
<tr>
<td>$B^+ \rightarrow \pi^0 \ell^+ \nu$ rejection</td>
<td>$</td>
</tr>
</tbody>
</table>

Table 4-3. Summary of event selection for all modes

**RECONSTRUCTION OF SIGNAL SIDE:** $B \rightarrow X \ell\nu$ SELECTION
Figure 4-10. Missing mass squared in the $\pi^0$ hypothesis for $B^+ \to \eta \ell^+\nu$ and $b \to u \ell \nu$ cross-feed events on Generic MC.

Although, some differences are observed, the overall agreement is good. Where differences are seen the induced systematic effects have been studied and they are discussed in Chapter 6.
Figure 4-11. Generic MC - data comparison for $B^+ \rightarrow \pi^0 \ell^+ \nu$ decay. The first column on the left displays the missing mass squared distribution after all cuts (top) and after only lepton cut (bottom). The second column shows the $\pi^0$ mass distribution. The third column shows the center of mass lepton momentum distribution. The fourth column shows the distribution of the energy for the most energetic photon. The ratio between data and MC is shown underneath each histogram.
Figure 4-12. Generic MC - data comparison for $B^{+} \rightarrow \pi^{0} \ell^{+}\nu$ and $B^{0} \rightarrow \pi^{-} \ell^{+}\nu$ decay. The first column on the left displays the total charge distribution after all cuts (top) and after only lepton cut (bottom) for $B^{+} \rightarrow \pi^{0} \ell^{+}\nu$ mode. The second column displays the missing mass squared distribution for $B^{0} \rightarrow \pi^{-} \ell^{+}\nu$ mode. The third column shows the center of mass lepton momentum distribution for $B^{0} \rightarrow \pi^{-} \ell^{+}\nu$ mode. The fourth shows the total charge distribution for $B^{0} \rightarrow \pi^{-} \ell^{+}\nu$ mode. The ratio between data and MC is shown underneath each histogram.
Figure 4-13. Generic MC - data comparison for $B^+ \rightarrow \eta \ell \nu$ decay. The first column on the left displays the missing mass squared distribution after only lepton cut (top) and after all cuts (bottom). The second column displays the $\eta$ mass distribution. The third column shows the center of mass lepton momentum distribution. The fourth shows the relative contribution of each reconstructed mode. The ratio between data and MC is shown underneath each histogram.
Figure 4-14. Generic MC - data comparison for $B^+ \rightarrow \tau^+ \ell^+ \nu$ decay. The first column on the left displays the missing mass squared distribution after only lepton cut (top) and after all cuts (bottom). The second column displays the $\tau^+$ mass distribution. The third column shows the center of mass lepton momentum distribution. The fourth shows the relative contribution of each reconstructed mode. The ratio between data and MC is shown underneath each histogram.
Figure 4-15. Generic MC - data comparison for $B^+ \rightarrow \eta' \ell^+ \nu$ decay. The first column on the left displays the number of charged track distribution after only lepton cut (top) and after all cuts (bottom). The second column displays the $\gamma$ momentum for $\eta' \rightarrow \rho^0 \gamma$. The third column shows the $\eta$ mass distributions for $\eta' \rightarrow \pi^+ \pi^-$. The fourth shows the $\rho$ mass distribution for $\eta' \rightarrow \rho^0 \gamma$. The ratio between data and MC is shown underneath each histogram.
Reconstruction of signal side: $B \rightarrow X \ell \nu$ selection

Alessia D’Orazio
This chapter outlines the overall strategy and the details to extract the exclusive branching fractions of the charmless semileptonic decays studied in this analysis: $B^+ \rightarrow \pi^0 \ell^+ \nu$, $B^0 \rightarrow \pi^- \ell^+ \nu$, $B^+ \rightarrow \eta \ell^+ \nu$ and $B^+ \rightarrow \eta' \ell^+ \nu$.

### 5.1 Branching Fraction Extraction

The measurement technique is based on the extraction of the ratio of branching ratios, $R_{\text{excl}/\text{sl}}$, given by:

$$R_{\text{excl}/\text{sl}} = \frac{\text{BR}(B \rightarrow (\pi^0, \pi, \eta, \eta') \ell \nu)}{\text{BR}(B \rightarrow X \ell \nu)} = \frac{N_{\text{true}}^\text{excl}}{N_{\text{true}}^\text{sl}} \quad (5.1)$$

where $\text{BR}(B \rightarrow X \ell \nu)$ is the total semileptonic branching fraction, containing both $b \rightarrow u \ell \nu$ and $b \rightarrow c \ell \nu$ events, and $\text{BR}(B \rightarrow (\pi^0, \pi, \eta, \eta') \ell \nu)$ is the exclusive branching fraction of one of the charmless semileptonic decays. $N_{\text{true}}^\text{excl}$ is the true number of signal events for a particular decay channel and $N_{\text{true}}^\text{sl}$ is the true number of semileptonic decays $B \rightarrow X \ell \nu$ events in the $B_{\text{reco}}$ sample.

The exclusive branching ratio of a particular channel is derived by multiplying the measured $R_{\text{excl}/\text{sl}}$ by the semileptonic branching ratio for charged or neutral $B$ decays. This approach is chosen to minimize systematic uncertainties, several of which tend to cancel in the ratio.

The number of observed events which contain a fully reconstructed $B$ meson and a charged lepton with a momentum $p^*_\ell$ above a given $p^*_\ell$ value (see Sec.4.3.1) is denoted as $N_{\text{meas}}^\text{excl}$. It can be related to the true number of $B \rightarrow X \ell \nu$ events before any requirement, $N_{\text{true}}^\text{sl}$, in the following way:

$$N_{\text{true}}^\text{sl} = \epsilon_{\text{sl}}^\text{excl} N_{\text{true}}^\text{excl} + B G_{\text{sl}} \quad (5.2)$$

where $B G_{\text{sl}}$ indicates the number of remaining background events, after the subtraction of the combinatorial background using the $m_E S$ fit (see Sec.5.1.1) and $\epsilon_{\text{sl}}^\text{excl}$ is the efficiency for selecting a lepton from a semileptonic $B$ decay in an event with a hadronic $B$ decay, reconstructed with tag efficiency $\epsilon_{\text{sl}}^\text{tag}$.

The number of signal events after the combinatoric background subtraction, $N_{\text{meas}}^\text{excl}$, and the number of peaking background events, $B G_{\text{excl}}$, are related to the true number of signal events, $N_{\text{true}}^\text{excl}$, by

$$N_{\text{true}}^\text{excl} = \frac{N_{\text{meas}}^\text{excl} - B G_{\text{excl}}}{\epsilon_{\text{excl}}^\text{excl} - \epsilon_{\text{excl}}^\text{sel}} \quad (5.3)$$

where $\epsilon_{\text{excl}}^\text{excl}$ is the efficiency for detecting exclusive charmless semileptonic decays on the recoil of a $B$ reconstructed with efficiency $\epsilon_{\text{excl}}^\text{sel}$, after applying the semileptonic selection that has efficiency $\epsilon_{\text{excl}}^\text{sel}$. In principle, the tag...
Table 5-1. Summary of the samples and the technique used to extract the input values to Eq. 5.4.

<table>
<thead>
<tr>
<th>Selection</th>
<th>extraction mode</th>
<th>sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{meas}_{excl}$</td>
<td>signal</td>
<td>data</td>
</tr>
<tr>
<td>$BG_{excl}$</td>
<td>signal</td>
<td>Fit to $m_{ES}$ and rescaling $Generic$ MC</td>
</tr>
<tr>
<td>$N^{meas}_{sl}$</td>
<td>semileptonic</td>
<td>data</td>
</tr>
<tr>
<td>$BG_{sl}$</td>
<td>semileptonic</td>
<td>$Generic$ MC</td>
</tr>
<tr>
<td>$\frac{e^{sl,excl}<em>l}{e^{sl,excl}</em>\ell}$</td>
<td>signal</td>
<td>MC</td>
</tr>
</tbody>
</table>

For the efficiency extraction, $\epsilon^{sl,excl}_l$ and $\epsilon^{sl,excl}_\ell$ should be the same for the two classes of events. Due to the difference in multiplicity and the different lepton momentum spectra, we expect $\epsilon^{sl,excl}_l$ and $\epsilon^{sl,excl}_\ell$ to be slightly different for the two classes of events.

The background contributions $BG_{sl}$ and $BG_{excl}$ and the efficiencies are estimated using the Monte Carlo simulation.

Finally, the ratio of branching ratios $R_{excl/sl}$ can then be determined as

$$R_{excl/sl} = \frac{BR(B \rightarrow \pi^0, \pi^-, \eta, \eta') e^{l} \ell \nu}{BR(B \rightarrow X \ell \nu)} = \frac{N^{meas}_{excl}}{N^{meas}_{sl}} \frac{(N^{meas}_{excl} - BG_{excl})/e^{excl}_{excl}}{(N^{meas}_{sl} - BG_{sl})} \times \frac{e^{sl}_{l,excl}}{e^{sl}_{\ell,excl}}.$$

Equation 5.4

Since, as already mentioned, $\epsilon^{sl,excl}_l$ and $\epsilon^{sl,excl}_\ell$ are slightly different, the ratio of efficiencies $\frac{e^{sl}_{l,excl}}{e^{sl}_{\ell,excl}}$ in Eq.5.4 is expected to be close to, but not equal to one.

Then, the branching fraction for each exclusive channel can be derived from the measured $R_{excl/sl}$, using the semileptonic branching ratio $BR(B \rightarrow X \ell \nu) = (10.73 \pm 0.28)%$ [59] and the ratio of the $B^0$ and $B^+$ lifetimes $\frac{\tau_{B^+}}{\tau_{B^0}} = 1.086 \pm 0.017$ [59].

The different techniques applied to extract the inputs to Eq. 5.4 are summarized in Tab. 5-1 and detailed in the following sections.

5.1.1 Determination of the semileptonic yield

$N^{meas}_{sl}$ is obtained requiring a high momentum lepton, $p^*_e > 0.5 \text{ GeV}/c$ or $p^*_\mu > 0.8 \text{ GeV}/c$, on the recoil of the $B_{reco}$. The full semileptonic selection has been already described in detail in Sec.4.3.1. An unbinned maximum likelihood fit to the $m_{ES}$ distribution on data (see Fig. 5-1) is performed to extract the signal and the combinatorial background contribution in the $m_{ES}$ signal region ($m_{ES} > 5.27 \text{ GeV}$) (see Sec.3.5.1). Any other residual background, $BG_{sl}$, mainly due to misidentified leptons and semileptonic charm decays, is estimated from the Monte Carlo simulation, scaled to the luminosity of the data sample by the ratio of the number of semileptonic events on data and Monte Carlo, and subtracted from $N^{meas}_{sl}$.

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5.1 Branching Fraction Extraction

5.1.2 Determination of the signal yield

For each exclusive decay channel under study, the signal selection criteria described in Sec. 4.3 are applied to the sample of semileptonic events to measure $N_{\text{excl}}^{\text{meas}}$. A fit to the $m_{ES}$ distribution on data (see Fig. 5-2 and 5-3) is performed to extract the number of events in the signal region ($m_{ES} > 5.27\, \text{GeV}$), which has to be properly subtracted from background in the $m_{ES}$ variable.

Two kinds of background are present in the $m_{ES}$ spectrum:

- **combinatoric background**, i.e. events for which the $B_{\text{reco}}$ candidate does not originate from a $B$ meson (continuum or $b\bar{b}$ combinatoric).

- **peaking background**, i.e. $b \rightarrow c\ell\nu$, $b \rightarrow u\ell\nu$ and non semileptonic events with a well reconstructed $B$ meson that mimic the signal in the recoil.

MEASUREMENT TECHNIQUE
The combinatorial background has been taken properly into account by extracting $N_{meas}^{\text{excl}}$ using an unbinned maximum likelihood fit to the $m_{ES}$ distribution, as shown in Fig. 5-2-5-3. Since the statistics is very small the signal shape parameters are fixed in the fit. They are have been obtained by fitting the data sample with looser cuts. On the contrary, since combinatoric background is strongly dependent on the cuts on the recoil, the background parameters are left floating in the fit. For the $B^0$ decays, the effect of B-mixing are included in the fit according to eq (4.3).

The peaking background present in the $m_{ES}$ signal region ($m_{ES} > 5.27 \text{ GeV}$) is separated in three contributions:

- $b \rightarrow c\ell\nu$ decays

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The number of peaking background events in the signal region can be written as:

$$BG^{sig}_{excl} = (N_u^{MC} C_u^{scale} + N_c^{MC} C_c^{scale} + N_{other}^{MC} C_{other}^{scale}) \cdot C^{norm}.$$  \hspace{1cm} (5.5)

$N_u^{MC}, N_c^{MC}$ and $N_{other}^{MC}$ are the number of background events in each category, after subtraction of the combinatoric background (as for $N_{excl}$), as estimated on Monte Carlo after all selection criteria have been applied. They are obtained using Signal Monte Carlo ($|V_{ub}|$ pure res. Generic vs. $b \rightarrow u \ell \nu$ exclusive in Tab. 2-7) for $N_u^{MC}$ and Generic Monte Carlo for $N_c^{MC}$ and $N_{other}^{MC}$.

$C_u^{scale}$, $C_c^{scale}$ and $C_{other}^{scale}$ represent the scaling factors and they correspond to the ratio of the number of semileptonic events in data and Monte Carlo. $C_c^{scale}$ is equal to $C_{other}^{scale}$ and $C_u^{scale}$ is computed as $C_u^{scale} = C_c^{scale} \cdot R_u$ where $R_u$, defined as the ratio of branching fractions for the exclusive $b \rightarrow u \ell \nu$ modes and the branching ratio for inclusive $B \rightarrow X \ell \nu$ expected from Monte Carlo, is used to take into account the ratio of branching fraction for the exclusive $b \rightarrow u \ell \nu$ modes we expect on data.

Then the number of peaking background events has been further rescaled to match the data in a $m_{miss}^2$ sideband region defined by $1 < m_{miss}^2 < 4 \text{ GeV}/c^2$ through the factor $C^{norm}$, that represent the ratio of the number of events in the sideband region on data and Generic Monte Carlo.

### 5.1.3 Efficiency corrections

The factor $\epsilon^{sl}_i/\epsilon^{excl}_i$ represents the ratio of the efficiencies for finding a full reconstructed $B_{reco}$ in events with $B \rightarrow (\pi^0, \pi, \eta, \eta') \ell \nu$ events in the recoil side and with $B \rightarrow X \ell \nu$ on the recoil side. The factor $\epsilon^{sl}_i/\epsilon^{excl}_i$ is the ratio of efficiencies of the lepton momentum cut for $B \rightarrow X \ell \nu$ and $B \rightarrow (\pi^0, \pi, \eta, \eta') \ell \nu$ decays, respectively. We evaluate the efficiency ratio $\epsilon^{sl}_i/\epsilon^{excl}_i$ on Generic MC simulation. The efficiencies $\epsilon^{excl}_i$ are determined from the Signal Monte Carlo sample.

The values of the ratio $\epsilon^{sl}_i/\epsilon^{excl}_i$ obtained for each decay mode are reported in Tab. 5-4.

### 5.2 Fit validations

In this section the validation of the fitting procedure using $b \rightarrow u \ell \nu$ (resonant)-Cocktail and Generic Monte Carlo samples is described.
5.2.1 Fit on *Cocktail Monte Carlo* sample

The full analysis has been run using the $b \to u \ell \nu$ (resonant)-*Cocktail* Monte Carlo sample, defined in Sec.2.9.2. The *Cocktail* Monte Carlo sample provides a sample with a high statistics for the signal, but it includes only a subset of the $B_{reco}$ modes. This high statistics test has been used to check if all signal efficiency factors are properly estimated. We recomputed $\epsilon_l^s/\epsilon_l^{exc}$ since $N_{sl}$ here is related to resonant $b \to u \ell \nu$ decays only. The fitting results for the ratio $R_{exc/sl}$ are in good agreement with the generated values, as shown in Tab.5-2.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$R_{exc/sl}(\text{resonant MC})$</th>
<th>$R_{true}^{exc/sl}(\text{resonantMC})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \pi^- \ell^+ \nu$</td>
<td>$0.237 \pm 0.009_{\text{stat}}$</td>
<td>0.245</td>
</tr>
<tr>
<td>$B^+ \to \pi^0 \ell^+ \nu$</td>
<td>$0.122 \pm 0.006_{\text{stat}}$</td>
<td>0.123</td>
</tr>
<tr>
<td>$B^+ \to \eta \ell^+ \nu$</td>
<td>$0.101 \pm 0.004_{\text{stat}}$</td>
<td>0.102</td>
</tr>
<tr>
<td>$B^+ \to \eta^\prime \ell^+ \nu$</td>
<td>$0.101 \pm 0.006_{\text{stat}}$</td>
<td>0.102</td>
</tr>
</tbody>
</table>

*Table 5-2. $R_{exc/sl}$ values from the fit on Cocktail - resonant and Monte Carlo sample, compared with generated values.*

5.2.2 Fit on *Generic Monte Carlo* sample

Another cross-check is to run the full analysis on the available *Generic* Monte Carlo sample. Since the *Generic* Monte Carlo should reproduce an unbiased data sample, this test is performed to check if the backgrounds are properly scaled. The equivalent statistics of the *Generic* sample corresponds to $\sim 1000 fb^{-1}$.

The fitting results for the ratio $R_{exc/sl}$ are in agreement with the generate values, as shown in Tab.5-3.

The projections of the fit results on the $m_{X_u}$ and $m_{miss}^2$ variables, including the extrapolated background components from Monte Carlo sample, are shown for each mode in Fig.5-4 and 5-5.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$R_{exc/sl}(\text{Generic MC}) \times 10^{-3}$</th>
<th>$R_{true}^{exc/sl}(\text{Generic MC}) \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to \pi^- \ell^+ \nu$</td>
<td>$1.75 \pm 0.13_{\text{stat}}$</td>
<td>1.66</td>
</tr>
<tr>
<td>$B^+ \to \pi^0 \ell^+ \nu$</td>
<td>$0.95 \pm 0.10_{\text{stat}}$</td>
<td>0.83</td>
</tr>
<tr>
<td>$B^+ \to \eta \ell^+ \nu$</td>
<td>$0.82 \pm 0.14_{\text{stat}}$</td>
<td>0.69</td>
</tr>
<tr>
<td>$B^+ \to \eta^\prime \ell^+ \nu$</td>
<td>$0.84 \pm 0.26_{\text{stat}}$</td>
<td>0.69</td>
</tr>
</tbody>
</table>

*Table 5-3. $R_{sl}^{exc}$ values from the fit on Generic Monte Carlo sample, compared with generated values.*
5.3 Results on data

The $R_{\text{excl}/sl}$ values obtained from the fit on the data sample are

- $\frac{BR(B^0 \to \pi^- \ell^+\nu)}{BR(B^0 \to X\nu)} = (0.99 \pm 0.23_{\text{stat}}) \times 10^{-3}$
- $\frac{BR(B^+ \to \pi^0 \ell^+\nu)}{BR(B^+ \to X\nu)} = (0.74 \pm 0.18_{\text{stat}}) \times 10^{-3}$
- $\frac{BR(B^+ \to \eta \ell^+\nu)}{BR(B^+ \to X\nu)} = (0.75 \pm 0.24_{\text{stat}}) \times 10^{-3}$
- $\frac{BR(B^+ \to \eta' \ell^+\nu)}{BR(B^+ \to X\nu)} = (0.30 \pm 0.53_{\text{stat}}) \times 10^{-3}$
Figure 5-5. Projection of the fit results on $m_{X_u}$ for Generic Monte Carlo sample. In each bin the total number of fitted signal and background events (dots) and the extrapolated background components (histograms) are shown for $B^+ \rightarrow \pi^0 \ell^+ \nu$ (top left), $B^+ \rightarrow \eta \ell^+ \nu$ (top right) and $B^+ \rightarrow \eta' \ell^+ \nu$ (bottom).

In Tab.5-4 the input parameters that enter in the calculation of the ratio $R_{excl/s}$ are summarized.

Fig.5-6 and 5-7 show the resulting data $m_{X_u}$ and $m_{miss}^2$ distributions for each mode. Signal and background components from the Monte Carlo, scaled to the number of event passing the semileptonic selection, are also overlaid.

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5.4 Measurement of partial Branching fractions for $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ decay modes

As explained in Chapter 1, $|V_{ub}|$ can be extracted from the measurement of the $B \to \pi\ell\nu$ decay rate. Indeed, under the assumption of massless leptons, the differential $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ decay rates are related to $|V_{ub}|$ by the Eq.

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |f_+(q^2)|^2 p_\pi^3,$$

where $p_\pi$ is the momentum of the pion in the rest frame of the $B$ meson, $q^2 = (p_\ell + p_\nu)^2$ is the invariant-mass squared of the lepton-neutrino system, and $f_+(q^2)$ is the hadronic form factor.

In this analysis $q^2$ is determined as $q^2 = (p_{T(4S)} - p_{B\text{reco}} - p_\pi)^2$, where $p_{T(4S)}$ is the sum of the four-momenta of the colliding beams, $p_{B\text{reco}}$ is measured four-momentum of the $B\text{reco}$ and $p_\pi$ is the measured four-momentum of $\pi$ or $\pi^0$.

The form factor $f_+(q^2)$ has been calculated so far with different theoretical models, already described in Sec.1.4. Since the Lattice QCD and LCSR calculations are reliable in limited regions of $q^2$, namely above 16 GeV/c$^2$ and below 16 GeV/c$^2$ respectively, we perform the measurement of the $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ differential decay rates as a function of $q^2$.

5.4.1 Analysis technique for measurement in $q^2$ bins

We consider three $q^2$ intervals: $q^2 < 8$ GeV$^2$, $8 < q^2 < 16$ GeV$^2$ and $q^2 > 16$ GeV$^2$.

After applying all selection cuts, we consider only events with a reconstructed $q^2$ lying in the interval under study. The efficiency and the signal and background components are estimated from Monte Carlo. In each bin the signal is defined as the $B^0 \to \pi^- \ell^+\nu$ ($B^+ \to \pi^0 \ell^+\nu$) events with a generated $q^2$ falling into the considered interval. In Fig. 5-8 the resolution on $q^2$, evaluated as the difference between the reconstructed and generated $q^2$ after all selection criteria, is shown for $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ events in the full $q^2$ range. Since the resolution is very good (about 0.25 GeV$^2$ for $B^0 \to \pi^- \ell^+\nu$ and 0.50 GeV$^2$ for $B^+ \to \pi^0 \ell^+\nu$...
Figure 5-6. Projection of the fit results on $m_{miss}^2$ for a data sample of 211 fb$^{-1}$ for $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ and 315 fb$^{-1}$ for $B^+ \rightarrow \eta \ell^+\nu$ and $B^+ \rightarrow \eta' \ell^+\nu$. In each bin the number of events from data (dots) and the extrapolated signal and background components from Monte Carlo (histograms) are shown for $B^+ \rightarrow \pi^0 \ell^+\nu$ (top left), $B^0 \rightarrow \pi^- \ell^+\nu$ (top right), $B^+ \rightarrow \eta \ell^+\nu$ (bottom left) and $B^+ \rightarrow \eta' \ell^+\nu$ (bottom right).

$B^+ \rightarrow \pi^0 \ell^+\nu$) compared to the width of the $q^2$ bins, the cross-feed among the different $q^2$ bins is small. This small amount of $B^0 \rightarrow \pi^- \ell^+\nu$ ($B^+ \rightarrow \pi^0 \ell^+\nu$) events generated outside that range are considered as background. The resolution on $q^2$ is also similar in the three $q^2$ regions as shown in Fig. 5-9 and Fig. 5-10 for $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ events, respectively. Then we can also neglect the correlation among the measurements in the different bins.

The partial branching fractions $\Delta BR(B^+ \rightarrow \pi^0 \ell^+\nu)$ and $\Delta BR(B^0 \rightarrow \pi^- \ell^+\nu)$ are measured with the same technique described in Sec. 5.1. The $m_{ES}$ fit parameters are fixed to the ones obtained in the whole $q^2$ analysis.

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5.4 Measurement of partial Branching fractions for $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ decay modes

5.4.2 Fit Results in $q^2$ bins

The results of the fit in three $q^2$ bins for the quantities used to determine the ratio of branching fractions are shown in Tab. 5-5 and Tab. 5-6, for $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ respectively.

In the tables, the results on the whole $q^2$ spectrum have been calculated as the sum of the results in the three $q^2$ bins. They are in good agreement with the results reported in Tab.5-4, obtained by running the fit on the whole $q^2$ spectrum, that have been used just as cross-check.

<table>
<thead>
<tr>
<th>$q^2$ bin [GeV$^2$/c$^4$]</th>
<th>$N_{\text{meas}}^{excl}$</th>
<th>$BG_{\text{excl}}$</th>
<th>$e_{\text{excl}}^{excl}$</th>
<th>$N_{\text{st}}^{meas} - BG_{\text{st}}$</th>
<th>$\frac{e_{\text{excl}}^{excl}}{N_{\text{meas}}^{excl}}$</th>
<th>$\frac{\Delta BR(B^0 \rightarrow \pi^- \ell^+ \nu)}{BR(B^0 \rightarrow X \ell \nu)}$ [x10$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$</td>
<td>7.0 ± 3.2</td>
<td>2.7 ± 1.6</td>
<td>0.70 ± 0.03</td>
<td>43800 ± 340</td>
<td>0.97 ± 0.08</td>
<td>0.08 ± 0.10</td>
</tr>
<tr>
<td>8 &lt; $q^2$ &lt; 16</td>
<td>10.5 ± 3.9</td>
<td>1.1 ± 1.0</td>
<td>0.54 ± 0.03</td>
<td>43800 ± 340</td>
<td>0.86 ± 0.09</td>
<td>0.32 ± 0.14</td>
</tr>
<tr>
<td>$q^2 &gt; 16$</td>
<td>18.8 ± 5.0</td>
<td>2.7 ± 1.6</td>
<td>0.57 ± 0.05</td>
<td>43800 ± 340</td>
<td>0.98 ± 0.15</td>
<td>0.63 ± 0.20</td>
</tr>
<tr>
<td>Total</td>
<td>36.3 ± 7.1</td>
<td>6.5 ± 2.3</td>
<td>-</td>
<td>43800 ± 340</td>
<td>-</td>
<td>1.03 ± 0.26</td>
</tr>
</tbody>
</table>

Table 5-5. Measurement of $R_{\text{excl/st}}$ for $B^0 \rightarrow \pi^- \ell^+ \nu$ in $q^2$ bins and corresponding inputs. The reported errors are statistical only.

<table>
<thead>
<tr>
<th>$q^2$ bin [GeV$^2$/c$^4$]</th>
<th>$N_{\text{meas}}^{excl}$</th>
<th>$BG_{\text{excl}}$</th>
<th>$e_{\text{excl}}^{excl}$</th>
<th>$N_{\text{st}}^{meas} - BG_{\text{st}}$</th>
<th>$\frac{e_{\text{excl}}^{excl}}{N_{\text{meas}}^{excl}}$</th>
<th>$\frac{\Delta BR(B^+ \rightarrow \pi^0 \ell^+ \nu)}{BR(B^+ \rightarrow X \ell \nu)}$ [x10$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$</td>
<td>7.7 ± 2.9</td>
<td>2.6 ± 1.6</td>
<td>0.44 ± 0.04</td>
<td>69600 ± 400</td>
<td>0.97 ± 0.12</td>
<td>0.15 ± 0.09</td>
</tr>
<tr>
<td>8 &lt; $q^2$ &lt; 16</td>
<td>13.5 ± 4.0</td>
<td>2.9 ± 1.7</td>
<td>0.42 ± 0.04</td>
<td>69600 ± 400</td>
<td>1.01 ± 0.09</td>
<td>0.35 ± 0.14</td>
</tr>
<tr>
<td>$q^2 &gt; 16$</td>
<td>12.9 ± 3.8</td>
<td>4.1 ± 2.0</td>
<td>0.37 ± 0.06</td>
<td>69600 ± 400</td>
<td>0.72 ± 0.13</td>
<td>0.24 ± 0.10</td>
</tr>
<tr>
<td>Total</td>
<td>34.1 ± 6.2</td>
<td>9.6 ± 3.1</td>
<td>-</td>
<td>69600 ± 400</td>
<td>-</td>
<td>0.74 ± 0.19</td>
</tr>
</tbody>
</table>

Table 5-6. Measurement of $R_{\text{excl/st}}$ for $B^+ \rightarrow \pi^0 \ell^+ \nu$ in $q^2$ bins and corresponding inputs. The reported errors are statistical only.

In Fig.5-11 and 5-12 the distributions in different $q^2$ bins of the $m^2_{\text{miss}}$ variable for data and extrapolated signal and background components from Monte Carlo are shown for $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$. 

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Figure 5-7. Projection of the fit results on $m_{X_s}$ for a data sample of 211 $fb^{-1}$ for $B^+ \rightarrow \pi^0 \ell^+\nu$ and 315 $fb^{-1}$ for $B^+ \rightarrow \eta \ell^+\nu$ and $B^+ \rightarrow \eta' \ell^+\nu$. In each bin the number of events from data (dots) and the extrapolated signal and background components from Monte Carlo (histograms) are shown for $B^+ \rightarrow \pi^0 \ell^+\nu$ (top left), $B^+ \rightarrow \eta \ell^+\nu$ (top right) and $B^+ \rightarrow \eta' \ell^+\nu$ (bottom).
5.4 Measurement of partial Branching fractions for $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ decay modes

Figure 5-8. Difference between reconstructed and generated $q^2$ for $B^0 \to \pi^- \ell^+\nu$ (left) and $B^+ \to \pi^0 \ell^+\nu$ (right) events after all cuts, using signal MC.

Figure 5-9. Difference between reconstructed and generated $q^2$ in each $q^2$ bin for $B^0 \to \pi^- \ell^+\nu$ events after all cuts, using signal Monte Carlo
Figure 5-10. Difference between reconstructed and generated $q^2$ in each $q^2$ bin for $B^+ \rightarrow \pi^0 \ell^+ \nu$ events after all cuts, using signal Monte Carlo.
5.4 Measurement of partial Branching fractions for $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ decay modes

Figure 5-11. Distribution of $m_{\text{miss}}^2$ for 211 $fb^{-1}$ of data (dots) and extrapolated signal and background components from Monte Carlo after all cuts for $B^0 \rightarrow \pi^- \ell^+ \nu$ in three $q^2$ bins: $q^2 < 8$ GeV$^2$ (top left), $8 < q^2 < 16$ GeV$^2$ (top right) and $q^2 > 16$ GeV$^2$ (bottom).
Figure 5-12. Distribution of $m_{\text{miss}}^2$ for $211 fb^{-1}$ of data (dots) and extrapolated signal and background components from Monte Carlo after all cuts for $B^+ \rightarrow \pi^0 \ell^+ \nu$ in three $q^2$ bins: $q^2 < 8 \text{GeV}^2$ (top left), $8 < q^2 < 16 \text{GeV}^2$ (top right) and $q^2 > 16 \text{GeV}^2$ (bottom).
Study of Systematic Uncertainties

In this analysis, different sources of systematic uncertainties have been taken into account: detector effects, bias in the fit, branching ratios estimates and theoretical uncertainties could affect the measurement and need to be studied.

In the following, the possible systematic effects that can impact the individual ingredients of the ratio of branching fractions measurement, extracted using the equation

\[ R_{\text{excl}/\text{sl}} = \frac{BR(B \rightarrow (\pi^0, \pi, \eta, \eta') \ell \nu)}{BR(B \rightarrow X \ell \nu)} = \frac{(N_{\text{excl}}^{\text{meas}} - BG_{\text{excl}})/\epsilon_{\text{sel}}^{\text{excl}}}{(N_{\text{sl}}^{\text{meas}} - BG_{\text{sl}})/\epsilon_{\text{sel}}^{\text{sl}}} \times \frac{\epsilon_{\ell}^{\text{sl}} \epsilon_{\ell}^{\text{excl}}}{\epsilon_{\ell}^{\text{excl}} \epsilon_{\ell}^{\text{sel}}}, \tag{6.1} \]

as described in Sec. 5.1, are summarized:

- \( N_{\text{sl}}^{\text{meas}} \): the number of events passing the semileptonic selection is the result of a fit to the \( m_{ES} \) distribution and it is therefore affected by its quality.

- \( BG_{\text{sl}} \): the dominant source of systematic uncertainty on the number of remaining background events after the semileptonic selection is affected by the quality of the simulation of the lepton misidentification. Since most of the background from secondary charm decays is already removed by the lepton charge-\( B \) flavor correlation, the remaining background comes from misidentified leptons.

- \( N_{\text{excl}} = N_{\text{excl}}^{\text{meas}} - BG_{\text{excl}} \): the number of signal events is derived in two steps: the number of events passing the exclusive selection (Sec.4.3.1) \( N_{\text{excl}}^{\text{meas}} \) is determined by a fit to the \( m_{ES} \) distribution and is sensitive to the quality the fit. Then, the number of peaking background events \( BG_{\text{excl}} \) is subtracted to \( N_{\text{excl}}^{\text{meas}} \) to determine \( N_{\text{excl}} \). Since \( BG_{\text{excl}} \) has been estimated from the Monte Carlo, it is sensitive to the Monte Carlo modeling for the particle identification, track and neutral reconstruction, the exclusive semileptonic branching fractions values and models for \( B \) and \( D \) mesons and baryons.

- \( \epsilon_{\ell}^{\text{sel}} \): the selection efficiency for the exclusive modes is extracted from the Monte Carlo simulation. It is sensitive to the quality of the simulation of the reconstruction of tracks and neutral particles. The two quantities that are most sensitive to the details of the event simulation are the missing mass squared, \( m_{\text{miss}}^{2} \), and the total charge of the event, \( Q_{\text{tot}} \).

- \( \epsilon_{\ell}^{\text{sl,excl}} \): the uncertainties in the lepton identification cancel to a large degree in the ratio of branching ratios, but the dependency on the correctness of the momentum spectra remains. The low threshold for accepting leptons (\( p^{*} > 0.5 \text{ GeV} \) for electrons and \( p^{*} > 0.8 \text{ GeV} \) for muons) keeps a very large fraction of the signal events and the theoretical uncertainty from the form factors gives a small contribution. Also the \( b \rightarrow c \ell \nu \) spectrum is affected by the theoretical model, but it has been measured by \( \text{BABAR} \) [?].
- $\epsilon_{i}^{sl,excl}$: A possible bias introduced by the selection of the Semi-exclusive reconstruction for the two classes of events could give rise to systematic effects.

The different sources of systematic uncertainties and their impact on the final results for each mode are reported in Tab.6-2 and 6-3 and described in detail in the following.

The total systematic uncertainties on the single decay modes are given by the sum in quadrature of all the individual contributions to the systematic uncertainties, that are assumed to be uncorrelated.

For the $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ decay modes, the estimation of systematic uncertainties for each $q^2$ bin is affected by low statistics in the Monte Carlo. Therefore, the systematic uncertainties have been calculated in the whole $q^2$ range and assumed to be the same in each of the three $q^2$ bins, except for those due to form factors and to Monte Carlo statistics. These uncertainties have been assessed separately for each $q^2$ bin.

### 6.1 Detector related systematics

The calculation of detector systematics uncertainties is largely affected by the lack of Monte Carlo statistics since we are dealing with very small effects. We divided the tracking and neutral reconstruction uncertainties in two separate factors: systematics on efficiency and systematics on background. Then we add the two effects in quadrature to quote the total systematic error associated to the reconstruction.

#### 6.1.1 Track reconstruction

Differences in the tracking efficiency and track resolution can impact the $m_{X_u}, m_{miss}^2$ and $Q_{tot}$ distributions.

The tracking efficiencies are well reproduced by the Monte Carlo simulations and the charged track spectrum shows a general agreement between data and Monte Carlo. The difference in data and Monte Carlo efficiency has been studied in detail using a control sample of $e^+e^- \rightarrow \tau^+\tau^-$ events, with one $\tau$ decaying leptonically and the other to three charged hadrons (plus an arbitrary number of neutrals) are used. They are a good control sample for this purpose because the $e^+e^- \rightarrow \tau^+\tau^-$ cross section is 0.94 $nb$ and the branching fraction to $\ell + 3$ hadrons is 11% so this sample allows large statistics tests. On the other hand the momentum distribution of tracks from $\tau$ decay is similar to that in $B$ decays. From this study we found that a flat 0.8% correction factor to the Monte Carlo is necessary to get a good data - Monte Carlo agreement and the systematic uncertainty on the track finding efficiency is 1.4% per track.

An estimate of the tracking systematics is then performed by removing randomly a fraction of tracks with a probability of 2.2% (sum in quadrature of 0.8% and 1.4%). The shift on the measured $R_{sl/excl}$ value with respect to the default one is quoted as systematic error.

**Alessia D’Orazio**
6.1 Detector related systematics

6.1.2 Particles identification

Lepton identification efficiencies and mis-identification probabilities are derived from different control samples. For electron efficiency, radiative Bhabha events are used. For pions the decay products of $K^0 \rightarrow \pi^+ \pi^-$ and three-prong $\tau$-decays are used. Muons with a momentum spectrum covering the range of interest are extracted from the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ channels. The resulting identification efficiencies and mis-identification probabilities in data are shown in Fig. 3-2 for electrons and Fig. 3-3 for muons. The statistical and systematic errors from data - Monte Carlo comparison are used to compute the systematic uncertainties.

The systematic uncertainties due to particle identification have been estimated by varying the electron and muon identification efficiency by $\pm 2\%$ and $\pm 3\%$, respectively and the relative mis-identification probabilities by $\pm 15\%$ before running the analysis. Then, the difference between the resulting $R_{x/s}$ and the default one is taken as systematic uncertainty.

6.1.3 Photon and $\pi^0$ reconstruction

Differences between data and Monte Carlo simulation in the photon detection efficiency and resolution, as well as additional energy depositions in the EMC, can impact the $L_5MNLPOQSGJG$ and $LPOQSRGJG$ distributions.

In order to check for disagreements between data and Monte Carlo simulation in efficiency and energy resolution a control sample of $\tau$ hadronic decays has been used. In fact, the $\tau$ hadronic decays represent an abundant source of $\pi^0$ s. $e^+e^- \rightarrow \tau^+\tau^-$ events are selected identifying the decay $\tau \rightarrow e\nu\bar{\nu}$. The recoiling $\tau$ is then studied. The ratio $R = N(\tau \rightarrow h^\pm \pi^0 )/N(\tau \rightarrow h^\pm \pi^0 \pi^0 )$ is computed both for data and Monte Carlo as a function of the $\pi^0$ energy in order to evaluate possible differences in efficiency. The agreement has been found to be good and the ratio is compatible with the unity in the full range. The resolution has been studied taking $\pi^0$ s from both $\tau \rightarrow h^\pm \pi^0$ and $\tau \rightarrow h^\pm \pi^0 \pi^0$ decays. The $\pi^0$ mass is fitted in energy bins and the resolution ($\sigma$ using a Gaussian fit) is then compared between data and Monte Carlo. The sample of $\pi^0$ s is also used to study the single photon efficiency.

As a consequence of this studies no efficiency correction is necessary on individual photons. A systematic uncertainty on the photon reconstruction of 1.8% per photon and an uncertainty on the $\pi^0$ reconstruction efficiency of 3.0% are assigned.

Moreover, we apply corrections for the energy scale, energy resolution and edge effect to the Monte Carlo to improve data and Monte Carlo agreement. The related systematic error is estimated by comparing the resulting $R_{excl/sl}$ obtained with (our default) and without these corrections.

STUDY OF SYSTEMATIC UNCERTAINTIES
6.2  $B_{reco}$ sample and fitting technique

6.2.1  $B^0 \leftrightarrow B^+$ cross-feed

The fact that the MC does not fully reproduce the data introduces possible differences in the $B_{reco}$ sample composition, in terms of correctly and incorrectly reconstructed modes and this effect can show up in a different amount of $B^0 \leftrightarrow B^+$ cross-feed (true $B^0$'s that have been reconstructed as $B^+$ and peak in the $m_{ES}$ variable). To evaluate the effect of $B^0$-$B^+$ cross-feed, the data events has been fitted using a Monte Carlo model without cross-feed ($B^\pm$ Generic MC only) and the difference with respect to the default result has been taken as systematic uncertainty.

6.2.2  Fit to the $m_{ES}$ distribution

The uncertainty of the $B_{reco}$ background subtraction is estimated by using an alternative approach to evaluate the number of semileptonic events $N_{sl}$, based on a binned $\chi^2$ fit, which is compared to the one obtained from the $m_{ES}$ fit to estimate the systematic error.

After applying the semileptonic selection, we consider the $m_{ES}$ distribution obtained from the data and from background components modeled with distributions taken from Monte Carlo simulation: $B^0\overline{B^0}$, $B^+B^-$ and $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$). In case the B meson is neutral, we consider two different samples: $B^0$ right sign ($Q_b(reco)Q_t < 0$) and $B^0$ wrong sign($Q_b(reco)Q_t > 0$).

The $m_{ES}$ value for data events is shifted run-by-run in order to reproduce the endpoint position of the Monte Carlo simulation and avoid biases in the background subtraction.

We fit the background normalization on data in the $m_{ES}$ sideband region, defined by $m_{ES} < 5.26$ GeV/$c^2$. The relative normalization of each component is determined by a binned $\chi^2$ fit. The $\chi^2$ function is defined as

$$\chi^2(C_{bkg}) = \sum_i \left( \frac{N_i^{meas} - C_{bkg}N_i^{bkg,MC}}{\sqrt{\delta N_i^{meas2} + \delta N_i^{MC2}}} \right)^2$$  \hspace{1cm} (6.2)

where $N_i^{meas}$ is the number of observed events in each i-th bin, $N_i^{bkg,MC}$ is the total background component, $C_{bkg}$ is the normalization of the background component, that is a free parameter, and $\delta N_i^{meas}$ and $\delta N_i^{MC}$ are the statistical errors for data and Monte Carlo respectively.

The normalization for $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) is fixed and the scaling factor is obtained from a comparison with off-peak data. Instead, the $B^+B^-$ and $B^0\overline{B^0}$ components and the normalization of the background component are floated in the fit. The total background contribution is then subtracted to the events in the $m_{ES}$ signal region ($m_{ES} > 5.27$ GeV/$c^2$) in order to extract the number of semileptonic events, separately for $B^0$ and $B^+$ and the difference is taken as a systematic error. For the neutral B we apply the mixing correction to obtain the total number of semileptonic events. The $m_{ES}$ distributions for $B^0$ and $B^+$, on 211 fb$^{-1}$ of data, are shown in Fig.6-1.

ALESSIA D’ORAZIO
Figure 6-1. $m_{ES}$ distributions for $B^0$ right sign (top left), $B^0$ wrong sign (top right) and $B^+$ (bottom) on a sample of 211 fb$^{-1}$ of data (dots) and rescaled background Monte Carlo components overlaid.
Study of Systematic Uncertainties

Figure 6-2. Lepton momentum spectrum (left) and $q^2$ spectrum (right) for ISGW2 model (black curve) and Ball04 model (red curve).

6.2.3 “Monte Carlo Statistics”

The finite available Monte Carlo statistics affects the measurement by introducing an uncertainty in the $m^2_{\text{miss}}$ shape of the background components. This is accounted for in the fit, where the pure statistical errors have been separated from the ones that can be associated to the Monte Carlo.

We sum in quadrature the statistical uncertainties on the ratio $\frac{\epsilon_{\text{incl}}}{\epsilon_{\text{excl}}} \frac{\epsilon_{\text{incl}}}{\epsilon_{\text{excl}}}$ and the selection efficiencies for the peaking background components ($b \rightarrow u\ell\nu$, $b \rightarrow c\ell\nu$ and other), estimated using the Generic Monte Carlo (see Sec. 5.1.3) and we take this value as systematic uncertainty associated to the “Monte Carlo statistics”.

6.3 Theoretical model dependence

The use of different theoretical models changes the shape of the lepton spectrum for the signal and, consequently, affects the efficiencies $\epsilon_{l}^{\text{incl}}$, $\epsilon_{l}^{\text{excl}}$ and $\epsilon_{\text{sel}}^{\text{excl}}$. The Monte Carlo sample used in this analysis has been generated using the ISGW2 model [17], as already explained in Sec.2.9.2. To evaluate the systematic associated to the theoretical model dependence, we reweighted the event distributions according to the recent calculations by Ball and Zwicky (Ball04) [19] based on light-cone sum rules. This calculation, among the ones available, result in distributions that differ most from those predicted by ISGW2, as shown in Fig.1-5. The lepton momentum and $q^2$ spectra for $B \rightarrow \pi \ell\nu$ for ISGW2 and Ball04 models are shown in Fig. 6-2.

We assign the efficiency variations with respect to ISGW2 model as systematic errors. This contribution is expected to be small because the selection efficiencies for each mode are almost flat over the phase space, as shown in Fig.6-3 and Fig.6-4 for $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ respectively.

For $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ decay modes the systematic uncertainties due to form factors have been calculated in each $q^2$ bin.

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6.4 Uncertainties on the Branching fractions of the background

In this Section the systematic effects arising from the uncertainties on the branching fractions of the background events for both $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_c \ell \nu$ decays are discussed separately.

6.4.1 Charm semileptonic Branching Ratios and charm decays

The exclusive semileptonic branching ratios for $B \rightarrow X_c \ell \nu$ decays and decay models for hadronic charm decays affect the $m_{X_u}$ shape of the $b \rightarrow c \ell \nu$ decays and thus the estimated background. Two systematic effects are considered:

- The individual branching fractions in the Monte Carlo simulations are known to differ from the world averages [59]. This difference has been accounted for by re-weighting the events to match the world averages.

STUDY OF SYSTEMATIC UNCERTAINTIES
The uncertainties in the measured branching ratios and the decay model introduce systematic errors. This effect has been estimated by determining $R_{\text{excl/sl}}$ for branching ratios that are varied within one standard deviation around the measured value of the branching fraction [59]. The spread in these results and the way it affects the $R_{\text{excl/sl}}$ measurement has been taken as the systematic error.

### 6.4.2 Charmless semileptonic Branching Ratios

Effects due to modeling of charmless semileptonic decays have been evaluated by varying the branching ratios for the exclusive charmless semileptonic decays taken from [59] within their known uncertainties, reported in Tab. 6-1.

<table>
<thead>
<tr>
<th>decay</th>
<th>variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to \pi \ell \nu$</td>
<td>30%</td>
</tr>
<tr>
<td>$B \to \rho \ell \nu$</td>
<td>30%</td>
</tr>
<tr>
<td>$B \to \omega \ell \nu$</td>
<td>30%</td>
</tr>
<tr>
<td>other</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Table 6-1.** Variations in the exclusive charmless branching ratios used to evaluate the systematics.

### 6.5 Background normalization for $B^+ \to \eta \ell^+ \nu$ and $B^+ \to \eta' \ell^+ \nu$

As already mentioned in Sec. 4.4, the data - Monte Carlo comparison for $B^+ \to \eta \ell^+ \nu$ and $B^+ \to \eta' \ell^+ \nu$ decay modes show an excess of events on the data around 1.5 GeV/c² in the distribution of the missing mass squared $m_{\text{miss}}^2$ after all the selection criteria, except the one on $m_{\text{miss}}^2$ have been applied (Fig. 4-13 and Fig. 4-14). Since the $m_{\text{miss}}^2$ sideband region is used to rescale the number of peaking background events, as described in detail in Sec. 5.1.2, the possible effects of the excess of events on the yield extraction should be taken into account.

We varied the sideband region definition used to normalize the background from $1 < m_{\text{miss}}^2 < 4$ GeV/c² to $1 < m_{\text{miss}}^2 < 2.5$ GeV/c², that corresponds to a variation on the number of background events $BG_{\text{excl}}$ of 11%. The difference on the $R_{\text{excl/sl}}$ value has been taken as systematic uncertainty.
<table>
<thead>
<tr>
<th>$B^0 \to \pi^- \ell^+ \nu$</th>
<th>$N_{sl}^{meas} - BG_{sl}$</th>
<th>$\epsilon_{sel}^{excl}$</th>
<th>$BG_{excl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron id</td>
<td>0.1 (%)</td>
<td>0.1 (%)</td>
<td>0.06 (ev.)</td>
</tr>
<tr>
<td>muon id</td>
<td>0.6 (%)</td>
<td>0.1 (%)</td>
<td>0.08 (ev.)</td>
</tr>
<tr>
<td>tracking efficiency</td>
<td>0.0 (%)</td>
<td>1.4 (%)</td>
<td>0.01 (ev.)</td>
</tr>
<tr>
<td>photon resolution</td>
<td>0.0 (%)</td>
<td>0.1 (%)</td>
<td>0.11 (ev.)</td>
</tr>
<tr>
<td>$B \to D \ell \nu X$ and $D$ BRs</td>
<td></td>
<td></td>
<td>0.05 (ev.)</td>
</tr>
<tr>
<td>$b \to u \ell \nu$ resonant</td>
<td></td>
<td></td>
<td>1.26 (ev.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B^+ \to \pi^0 \ell^+ \nu$</th>
<th>$N_{sl}^{meas} - BG_{sl}$</th>
<th>$\epsilon_{sel}^{excl}$</th>
<th>$BG_{excl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron id</td>
<td>0.1 (%)</td>
<td>0.3 (%)</td>
<td>0.10 (ev.)</td>
</tr>
<tr>
<td>muon id</td>
<td>0.7 (%)</td>
<td>0.2 (%)</td>
<td>0.86 (ev.)</td>
</tr>
<tr>
<td>tracking efficiency</td>
<td>0.0 (%)</td>
<td>0.0 (%)</td>
<td>0.32 (ev.)</td>
</tr>
<tr>
<td>photon resolution</td>
<td>0.0 (%)</td>
<td>2.2 (%)</td>
<td>0.47 (ev.)</td>
</tr>
<tr>
<td>$B \to D \ell \nu X$ and $D$ BRs</td>
<td></td>
<td></td>
<td>0.64 (ev.)</td>
</tr>
<tr>
<td>$b \to u \ell \nu$ resonant</td>
<td></td>
<td></td>
<td>0.43 (ev.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B^+ \to \eta^+ \nu$</th>
<th>$N_{sl}^{meas} - BG_{sl}$</th>
<th>$\epsilon_{sel}^{excl}$</th>
<th>$BG_{excl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron id</td>
<td>0.0 (%)</td>
<td>6.3 (%)</td>
<td>0.37 (ev.)</td>
</tr>
<tr>
<td>muon id</td>
<td>0.4 (%)</td>
<td>6.9 (%)</td>
<td>0.54 (ev.)</td>
</tr>
<tr>
<td>tracking efficiency</td>
<td>0.0 (%)</td>
<td>9.0 (%)</td>
<td>0.50 (ev.)</td>
</tr>
<tr>
<td>neutral corrections</td>
<td>0.0 (%)</td>
<td>7.6 (%)</td>
<td>0.82 (ev.)</td>
</tr>
<tr>
<td>$B \to D \ell \nu X$ and $D$ BRs</td>
<td></td>
<td></td>
<td>1.20 (ev.)</td>
</tr>
<tr>
<td>$b \to u \ell \nu$ resonant</td>
<td></td>
<td></td>
<td>0.48 (ev.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B^+ \to \eta' \ell^+ \nu$</th>
<th>$N_{sl}^{meas} - BG_{sl}$</th>
<th>$\epsilon_{sel}^{excl}$</th>
<th>$BG_{excl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron id</td>
<td>0.1 (%)</td>
<td>3.8 (%)</td>
<td>0.16 (ev.)</td>
</tr>
<tr>
<td>muon id</td>
<td>0.4 (%)</td>
<td>4.1 (%)</td>
<td>0.19 (ev.)</td>
</tr>
<tr>
<td>tracking efficiency</td>
<td>0.0 (%)</td>
<td>4.2 (%)</td>
<td>0.16 (ev.)</td>
</tr>
<tr>
<td>neutral corrections</td>
<td>0.0 (%)</td>
<td>2.7 (%)</td>
<td>2.30 (ev.)</td>
</tr>
<tr>
<td>$B \to D \ell \nu X$ and $D$ BRs</td>
<td></td>
<td></td>
<td>1.41 (ev.)</td>
</tr>
<tr>
<td>$b \to u \ell \nu$ resonant</td>
<td></td>
<td></td>
<td>0.46 (ev.)</td>
</tr>
</tbody>
</table>

Table 6-2. Breakdown of the systematic uncertainties in the different terms used to calculate $R_{excl}$/sl.

STUDY OF SYSTEMATIC UNCERTAINTIES
### Table 6-3. Systematic uncertainties in the measurement of $R_{excl/st}$. The total systematic uncertainties on the single decay modes are given by the sum in quadrature of all the individual contributions. Note that the systematic uncertainties on $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ have been expressed as relative uncertainties on $R_{excl/st}$. Instead the systematic uncertainties on $B^+ \rightarrow \eta\ell^+\nu$ and $B^+ \rightarrow \eta'\ell^+\nu$ have been expressed as absolute uncertainties on $R_{excl/st}$.

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>$B^0 \rightarrow \pi^- \ell^+\nu$</th>
<th>$B^+ \rightarrow \pi^0 \ell^+\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron ID</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Muon ID</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Tracking Efficiency</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Photon resolution, $\pi^0$ reco</td>
<td>1.2</td>
<td>3.7</td>
</tr>
<tr>
<td>$m_{ES}$ fit</td>
<td>5.7</td>
<td>2.7</td>
</tr>
<tr>
<td>Cross-feed $B^0 \leftrightarrow B^+$</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Form-factor model dependence $(q^2 &lt; 8)$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Form-factor model dependence $(q^2 &gt; 8, q^2 &lt; 16)$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Form-factor model dependence $(q^2 &gt; 16)$</td>
<td>0.1</td>
<td>2.2</td>
</tr>
<tr>
<td>$B \rightarrow Dl\nu X$ and $D$ BRs</td>
<td>0.2</td>
<td>2.6</td>
</tr>
<tr>
<td>$b \rightarrow u\ell\nu$ resonant</td>
<td>4.2</td>
<td>1.7</td>
</tr>
<tr>
<td>MC statistics $(q^2 &lt; 8)$</td>
<td>18.3</td>
<td>19.8</td>
</tr>
<tr>
<td>MC statistics $(8 &lt; q^2 &lt; 16)$</td>
<td>11.8</td>
<td>14.7</td>
</tr>
<tr>
<td>MC statistics $(q^2 &gt; 16)$</td>
<td>17.6</td>
<td>23.0</td>
</tr>
<tr>
<td>Total error $(q^2 &lt; 8)$</td>
<td>21.2</td>
<td>21.2</td>
</tr>
<tr>
<td>Total error $(q^2 &gt; 16)$</td>
<td>16.0</td>
<td>16.7</td>
</tr>
<tr>
<td>Total error $(8 &lt; q^2 &lt; 16)$</td>
<td>20.6</td>
<td>24.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>$B^+ \rightarrow \eta\ell^+\nu$</th>
<th>$B^+ \rightarrow \eta'\ell^+\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron ID</td>
<td>±0.03</td>
<td>±0.01</td>
</tr>
<tr>
<td>Muon ID</td>
<td>±0.03</td>
<td>±0.02</td>
</tr>
<tr>
<td>Tracking Efficiency</td>
<td>±0.04</td>
<td>±0.02</td>
</tr>
<tr>
<td>Photon resolution, $\pi^0$ reco</td>
<td>±0.03</td>
<td>±0.03</td>
</tr>
<tr>
<td>$m_{ES}$ fit</td>
<td>±0.09</td>
<td>±0.04</td>
</tr>
<tr>
<td>Cross-feed $B^0 \leftrightarrow B^+$</td>
<td>±0.01</td>
<td>±0.09</td>
</tr>
<tr>
<td>Theoretical model dependence</td>
<td>±0.03</td>
<td>±0.02</td>
</tr>
<tr>
<td>$B \rightarrow Dl\nu X$ and $D$ BRs</td>
<td>±0.04</td>
<td>±0.12</td>
</tr>
<tr>
<td>$b \rightarrow u\ell\nu$ resonant</td>
<td>±0.02</td>
<td>±0.05</td>
</tr>
<tr>
<td>Background normalization</td>
<td>±0.08</td>
<td>±0.07</td>
</tr>
<tr>
<td>MC statistics</td>
<td>±0.12</td>
<td>±0.20</td>
</tr>
<tr>
<td>Total error</td>
<td>±0.19</td>
<td>±0.27</td>
</tr>
</tbody>
</table>
Branching Fractions and $|V_{ub}|$ extraction

In this chapter we report the final results for the exclusive charmless semileptonic decays branching fractions measurements and the extraction of $|V_{ub}|$.

The exclusive branching fraction measurements has been derived from the measured $R_{excl/sl}$ by including the systematic uncertainties quoted in Tab.6-3 and by multiplying $R_{excl/sl}$ for the values of $BR(B^0 \rightarrow X\ell\nu)$ or $BR(B^\pm \rightarrow X\ell\nu)$. The $BR(B^0 \rightarrow X\ell\nu)$ and $BR(B^\pm \rightarrow X\ell\nu)$ values have been derived starting from the inclusive semileptonic branching ratio $BR(B \rightarrow X\ell\nu) = (10.73 \pm 0.28)\%$ and using the ratio of the $B^0$ and $B^+$ lifetimes $\frac{\tau_{B^+}}{\tau_{B^0}} = 1.086 \pm 0.017$ [59], $BR(B^0 \rightarrow X\ell\nu)$ and $BR(B^\pm \rightarrow X\ell\nu)$ can be calculated as

$$BR(B^0 \rightarrow X\ell\nu) = BR(B \rightarrow X\ell\nu) \frac{2}{1 + \frac{\tau_{B^+}}{\tau_{B^0}}}$$

$$BR(B^\pm \rightarrow X\ell\nu) = BR(B^0 \rightarrow X\ell\nu) \frac{\tau_{B^+}}{\tau_{B^0}}.$$ (7.1) (7.2)

7.1 $B^+ \rightarrow \eta\ell^+\nu$ and $B^+ \rightarrow \eta'\ell^+\nu$ branching fractions

The measured exclusive branching fractions for $B^+ \rightarrow \eta\ell^+\nu$ and $B^+ \rightarrow \eta'\ell^+\nu$ on a data sample of 315 $fb^{-1}$ are:

$$BR(B^+ \rightarrow \eta\ell^+\nu) = (0.84 \pm 0.27_{\text{stat}} \pm 0.21_{\text{syst}}) \times 10^{-4},$$ (7.3)

$$BR(B^+ \rightarrow \eta'\ell^+\nu) = (0.33 \pm 0.60_{\text{stat}} \pm 0.30_{\text{syst}}) \times 10^{-4}$$ (7.4)

where the first error is statistical and the second error include all systematic and theoretical uncertainties.

The resulting significance is 2.55$\sigma$ for $B^+ \rightarrow \eta\ell^+\nu$ and 0.95$\sigma$ for $B^+ \rightarrow \eta'\ell^+\nu$. Since the yields for the $B^+ \rightarrow \eta\ell^+\nu$ and $B^+ \rightarrow \eta'\ell^+\nu$ decay modes are not statistically significant, we set un upper limit for these branching fractions. The significance and the upper limit has been calculated including all the systematic and statistical uncertainties on the background, as explained in the following.

The upper limits are calculated using a frequentist approach by means of a toy Monte Carlo.

We consider as input the following quantities:

- $N_{DATA}$ is the number of events extracted from the data.
- $N_{BKG}$ is the sum of the number of peaking and combinatorial background events.

- $BR_{EXCL}$ is the measured branching fraction for the signal decay mode. Then this quantity is varied in order to get the upper limit, as explained in the follow.

- $BR_{FACT}$ is defined by $\frac{BR_{EXCL}}{N_{DATA} - N_{BKG}}$ and it gives an estimation of the number of $B_{rec}$ semileptonic events.

- $\sigma(N_{BKG})$ is the error associated to $N_{BKG}$. It is given by the sum in quadrature of the statistical errors associated to the number of peaking and combinatorial background events (taken from the fit) and the systematic uncertainties associated to the background events. We consider as systematic uncertainties on the background all the values quoted in Tab 6-3, except for the error on efficiency, theoretical model and inclusive branching fraction, which are assigned to the number of signal events.

- $\sigma(N_{SIG})$ is the error associated to the number of signal events. It is given by the sum in quadrature of the systematic uncertainties assigned to the signal, that are the error on the efficiency, the error associated to the theoretical model and the error on the inclusive branching fractions.

For a given $BR_{EXCL}$, the number of background events has been fluctuated according to a Gaussian distribution with mean value $N_{BKG}$ and standard deviation $\sigma(N_{BKG})$ and the number of signal events has been fluctuated according to a Gaussian distribution with mean value $\frac{BR_{EXCL}}{BR_{FACT}}$ and standard deviation $\sigma(N_{SIG})$. The number of observed events, defined by the sum of Gaussian distributions smeared the signal and background events, is distributed as a Poissonia probability density function. The fraction $\alpha$ of the cases in which the number of observed events, extracted from the Poisson distribution, is greater or equal than the total number of observed events in data $N_{DATA}$ has been calculated.

A scan on the possible values for $BR_{EXCL}$ has been made and the corresponding values of $\alpha$ have been computed. The upper limit at 90% of confidence level has been defined as the value of $BR_{EXCL}$ corresponding to $\alpha = 90\%$.

We get the following 90% confidence level (C. L.) upper limits:

$$BR(B^+ \rightarrow \eta\ell^+\nu) < 1.4 \times 10^{-4}(90\%\,C.L.) \quad (7.5)$$

$$BR(B^+ \rightarrow \eta^\prime\ell^+\nu) < 1.3 \times 10^{-4}(90\%\,C.L.) \quad (7.6)$$

### 7.2 $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ branching fractions

The $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ partial branching fractions measured on a data sample of 211 $fb^{-1}$ are reported in Tab. 7-1 and Tab. 7-2 respectively, and shown in Fig. 7-1 as function of $q^2$.

The measurements of $BR(B^0 \rightarrow \pi^- \ell^+\nu)$ and $BR(B^+ \rightarrow \pi^0 \ell^+\nu)$ can be combined assuming the isospin symmetry relation

$$\Gamma(B \rightarrow \pi\ell\nu) \equiv \Gamma(B^0 \rightarrow \pi^- \ell^+\nu) = 2 \times \Gamma(B^+ \rightarrow \pi^0 \ell^+\nu). \quad (7.7)$$
Using the ratio of the $B^0$ and $B^+$ lifetimes $\frac{\tau_{B^+}}{\tau_{B^0}} = 1.086 \pm 0.017$, the relationship between the branching fractions is

$$BR(B^0 \to \pi^- \ell^+\nu) = \frac{2 \tau_{B^0}}{\tau_{B^+}} BR(B^+ \to \pi^0 \ell^+\nu) = (1.84 \pm 0.03) BR(B^+ \to \pi^0 \ell^+\nu). \quad (7.8)$$

We average the six results for the partial branching fractions by following the prescription suggested by the Heavy Flavor Averaging Group in [67]. We make the following assumptions in combining the measurements:

- The statistical errors are uncorrelated.
- The systematic errors related to the Monte Carlo statistics and the charm and charmless branching ratios are considered uncorrelated.
- Other systematic errors are fully correlated.

We must also take into account how the measurement errors scale when the central values are varied to avoid bias in the average. We assume that the statistical errors scale with the square root of the signal branching fraction. This is a reasonable assumption when the signal is cleanly separated from the background. Since almost all the systematic errors are multiplicative, we assume that they scale linearly with the signal branching fraction. Combining the $B^0 \to \pi^- \ell^+\nu$ and $B^+ \to \pi^0 \ell^+\nu$ channels, we find the partial and total branching fractions reported in Tab.7-3 and shown in Fig. 7-2.

<table>
<thead>
<tr>
<th>$q^2$ bin</th>
<th>$\Delta BR(B^0 \to \pi^- \ell^+\nu) [10^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$ GeV</td>
<td>$0.09 \pm 0.01_{\text{stat}} \pm 0.02_{\text{syst}}$</td>
</tr>
<tr>
<td>$8 &lt; q^2 &lt; 16$ GeV</td>
<td>$0.33 \pm 0.15_{\text{stat}} \pm 0.05_{\text{syst}}$</td>
</tr>
<tr>
<td>$q^2 &gt; 16$ GeV</td>
<td>$0.65 \pm 0.20_{\text{stat}} \pm 0.12_{\text{syst}}$</td>
</tr>
<tr>
<td>Total</td>
<td>$1.07 \pm 0.27_{\text{stat}} \pm 0.15_{\text{syst}}$</td>
</tr>
</tbody>
</table>

Table 7-1. Partial and total branching ratio results for $B^0 \to \pi^- \ell^+\nu$ decay mode.

<table>
<thead>
<tr>
<th>$q^2$ bin</th>
<th>$\Delta BR(B^+ \to \pi^0 \ell^+\nu) [10^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$ GeV</td>
<td>$0.16 \pm 0.11_{\text{stat}} \pm 0.04_{\text{syst}}$</td>
</tr>
<tr>
<td>$8 &lt; q^2 &lt; 16$ GeV</td>
<td>$0.39 \pm 0.16_{\text{stat}} \pm 0.06_{\text{syst}}$</td>
</tr>
<tr>
<td>$q^2 &gt; 16$ GeV</td>
<td>$0.26 \pm 0.12_{\text{stat}} \pm 0.06_{\text{syst}}$</td>
</tr>
<tr>
<td>Total</td>
<td>$0.82 \pm 0.22_{\text{stat}} \pm 0.11_{\text{syst}}$</td>
</tr>
</tbody>
</table>

Table 7-2. Partial and total branching ratio results for $B^+ \to \pi^0 \ell^+\nu$ decay mode.
As seen in detail in Sec. 1.3, $|V_{ub}|$ can be measured from the rate of the exclusive charmless semileptonic decays.

Integrating Eq. (1.43) in a given $q^2$ interval, $q_{min}^2 < q^2 < q_{max}^2$, $|V_{ub}|$ can be extracted from the measured partial branching fractions $\Delta BR(B \rightarrow \pi \ell \nu)$ using the following expression:

$$|V_{ub}| = \sqrt{\frac{\Delta BR(B^0 \rightarrow \pi^- \ell^+ \nu)}{\Delta \zeta \cdot \tau_{B^0}}}$$

where $\tau_{B^0} = 1.532 \pm 0.014 \text{ps}^{-1}$ [59] is the $B^0$ lifetime, $\Delta BR(B^0 \rightarrow \pi^- \ell^+ \nu)$ is the combined partial branching ratio for a given $q^2$ interval and $\Delta \zeta$ is the predicted form-factor normalization for the same $q^2$, defined by Eq. (1.45).
As discussed in Sec. 1.4, there are different theoretical calculations of \( f_+ (q^2) \) available in literature, which predict different \( q^2 \) spectra and which are reliable in different \( q^2 \) ranges.

Two methods can be used for the extraction of \( |V_{ub}| \). We can measure \( |V_{ub}| \) from the total branching fraction measurements, but this introduce a systematic error associated to the extrapolation of the form-factors, or from the partial branching fractions, that don’t have extrapolation uncertainties, but the statistical errors are higher than the other method.

We used Light Cone Sum Rules (LCSR) calculations [19, 20] in the range \( q^2 < 16 \text{ GeV}^2/c^2 \) and the unquenched lattice QCD (LQCD) calculations [24, 23] (HPQCD and FNAL in Tab. 7-4) in the range \( q^2 > 16 \text{ GeV}^2/c^2 \) The extrapolation of the form-factors to the full \( q^2 \) range is performed by using empirical functions but introduces additional uncertainties which must be taken into account.

Tab. 7-4 summarizes the values of \( |V_{ub}| \) that we extracted from the measured \( B^0 \to \pi^- \ell^+\nu \) partial and total branching fractions in Tab.7-3.

### 7.4 Combined \( B \to \pi\ell\nu \) measurements with Tagged B Mesons

The results of this analysis have been combined with the BABAR measurements which use semileptonic tagged events [43]. This technique is very similar to the hadronic tag technique. The tagging \( B \) meson is reconstructed in the semileptonic decay \( B \to D^{(*)}\ell\nu \) and the signature of a \( B \to \pi\ell\nu \) decay is searched in the recoiling system.
Table 7-4. Values of $|V_{ub}|$ derived from the combined partial branching ratios (first three rows) and the combined total branching ratio (last three rows) using the different form factor calculations. The first two errors on $|V_{ub}|$ come from the statistical and systematic uncertainties of the branching ratio measurements. The third errors correspond to the uncertainties on $\Delta \zeta$ due to the form factor calculations, and are taken from Refs. [20, 24, 23].

| FF calculation | $q^2$ range | $\Delta \zeta \, (ps^{-1})$ | $|V_{ub}| \, (10^{-3})$ |
|----------------|-------------|--------------------------|--------------------------|
| Ball-Zwicky [20] | $< 16 \text{ GeV}^2/c^4$ | 5.44 ± 1.43 | 2.8 ± 0.5_{\text{stat}} ± 0.2_{\text{syst}} ±^{+0.5}_{-0.3_{\text{FF}}}$ |
| HPQCD [24] | $> 16 \text{ GeV}^2/c^4$ | 1.29 ± 0.32 | 5.4 ± 0.7_{\text{stat}} ± 0.4_{\text{syst}} ±^{+0.8}_{-0.6_{\text{FF}}}$ |
| FNAL [23] | $> 16 \text{ GeV}^2/c^4$ | 1.83 ± 0.50 | 4.6 ± 0.6_{\text{stat}} ± 0.4_{\text{syst}} ±^{+0.8}_{-0.5_{\text{FF}}}$ |
| Ball-Zwicky [20] | full | 7.74 ± 2.32 | 3.2 ± 0.3_{\text{stat}} ± 0.2_{\text{syst}} ±^{+0.6}_{-0.4_{\text{FF}}}$ |
| HPQCD [24] | full | 5.70 ± 1.71 | 3.8 ± 0.4_{\text{stat}} ± 0.2_{\text{syst}} ±^{+0.7}_{-0.5_{\text{FF}}}$ |
| FNAL [23] | full | 6.24 ± 2.12 | 3.6 ± 0.4_{\text{stat}} ± 0.2_{\text{syst}} ±^{+0.8}_{-0.5_{\text{FF}}}$ |

Table 7-5. Measured partial branching fractions (in $10^{-4}$) for $B \rightarrow \pi \ell \nu$ using the semileptonic tag (first two rows) and the hadronic tag (last two rows) technique. The first error is statistical and the second one systematic.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$q^2 &lt; 8 \text{ GeV}^2/c^2$</th>
<th>$8 &lt; q^2 &lt; 16 \text{ GeV}^2/c^2$</th>
<th>$q^2 &gt; 16 \text{ GeV}^2/c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$ s.l. tag</td>
<td>0.498 ± 0.163 ± 0.050</td>
<td>0.333 ± 0.140 ± 0.035</td>
<td>0.285 ± 0.150 ± 0.039</td>
</tr>
<tr>
<td>$B^+ \text{ s.l. tag}$</td>
<td>0.182 ± 0.084 ± 0.018</td>
<td>0.452 ± 0.134 ± 0.049</td>
<td>0.096 ± 0.115 ± 0.042</td>
</tr>
<tr>
<td>$B^0 \text{ had. tag}$</td>
<td>0.086 ± 0.107 ± 0.017</td>
<td>0.333 ± 0.147 ± 0.048</td>
<td>0.653 ± 0.202 ± 0.127</td>
</tr>
<tr>
<td>$B^+ \text{ had. tag}$</td>
<td>0.164 ± 0.105 ± 0.034</td>
<td>0.392 ± 0.155 ± 0.064</td>
<td>0.263 ± 0.117 ± 0.063</td>
</tr>
</tbody>
</table>

We take weighted averages of the measured partial branching fractions in each $q^2$ bin. In combining the measurements, we use the procedure suggested by the Heavy Flavor Averaging Group [67], already described in Sec.7.2. Again, we assume that the statistical errors are uncorrelated. The systematic errors are assumed to be fully correlated, except for:

- The Monte Carlo statistical errors are completely uncorrelated.
- The errors related to the tagging ($B_{\text{tag}}$) and the fitting methods are uncorrelated between the semileptonic-tag and hadronic-tag analyses.

Most of the systematic errors are multiplicative and scale linearly with the signal branching fractions. The exceptions are:

- the $B \rightarrow X_c \ell \nu$ background systematics (branching fractions),
- the $B \rightarrow X_\tau \ell \nu$ background systematics (branching fractions, form factors, shape function),

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7.5 Discussion of the results

The results for the $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$ branching fractions obtained in this analysis are in agreement with the other measurements available on $B \to \pi \ell \nu$, that are summarized in Tab.7-8 and also displayed in Fig. 7-3. The measurements for the $B^+ \to \pi^0 \ell^+ \nu$ mode, reported in Fig. 7-3, have been multiplied by a factor of two to reflect isospin expectations, and corrected for the difference in lifetimes of $B$ mesons.

### Table 7-6

<table>
<thead>
<tr>
<th>$q^2 &lt; 8\text{ GeV}^2$</th>
<th>$8 &lt; q^2 &lt; 16\text{ GeV}^2$</th>
<th>$q^2 &gt; 16\text{ GeV}^2$</th>
<th>All $q^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$</td>
<td>$0.383 \pm 0.121 \pm 0.036$</td>
<td>$0.333 \pm 0.101 \pm 0.030$</td>
<td>$0.474 \pm 0.129 \pm 0.059$</td>
</tr>
<tr>
<td>$B^+$</td>
<td>$0.176 \pm 0.066 \pm 0.018$</td>
<td>$0.429 \pm 0.102 \pm 0.043$</td>
<td>$0.218 \pm 0.091 \pm 0.047$</td>
</tr>
<tr>
<td>Combined</td>
<td>$0.355 \pm 0.086 \pm 0.029$</td>
<td>$0.518 \pm 0.097 \pm 0.041$</td>
<td>$0.457 \pm 0.104 \pm 0.056$</td>
</tr>
</tbody>
</table>

The combination is done using an iterative $\chi^2$-minimization procedure. The results are summarized in Table 7-6.

We also test the isospin symmetry by computing the difference

$$\Delta BR(B^0 \to \pi^- \ell^+ \nu) - 2 \frac{T_{B^0}}{T_{B^+}} \Delta BR(B^+ \to \pi^0 \ell^+ \nu)$$

in each $q^2$ bin and divide by the uncertainty. We find $0.34\sigma$, $-2.22\sigma$, and $0.30\sigma$ in the three $q^2$ bins, respectively. A $\chi^2$ value can be computed as $0.34^2 + 2.22^2 + 0.30^2 = 5.15$ for 3 degrees of freedom, which translates to a 16% probability. For the combined result, we find the total $\chi^2$ to be 10.2 for 9 degrees of freedom. The $\chi^2$ probability is 34%.

We derived the $|V_{ub}|$ values from the partial or total decay rates, using different theoretical calculations:

- for partial $\Delta BR(q^2 > 16\text{ GeV}^2)$ we used the unquenched lattice QCD calculations by HPQCD [25] and FNAL [23], and the quenched QCD calculation by APE [68].
- for the partial $\Delta BR(q^2 < 16\text{ GeV}^2)$ we used the latest LCSR calculation by Ball and Zwicky [20].
- for the total $BR$ we used Ball04, HPQCD, FNAL, and APE calculations.

The values of $\Delta \zeta$ and their uncertainties have been supplied by the authors of [20, 25, 23, 68]. We then calculated $|V_{ub}|$ values using the Eq.7.9.

Tab. 7-7 summarizes the values of $|V_{ub}|$ extracted from the partial and total branching fraction measurements with the tagged $B$ mesons.

### 7.5 Discussion of the results

We also test the isospin symmetry by computing the difference

$$\Delta BR(B^0 \to \pi^- \ell^+ \nu) - 2 \frac{T_{B^0}}{T_{B^+}} \Delta BR(B^+ \to \pi^0 \ell^+ \nu)$$

in each $q^2$ bin and divide by the uncertainty. We find $0.34\sigma$, $-2.22\sigma$, and $0.30\sigma$ in the three $q^2$ bins, respectively. A $\chi^2$ value can be computed as $0.34^2 + 2.22^2 + 0.30^2 = 5.15$ for 3 degrees of freedom, which translates to a 16% probability. For the combined result, we find the total $\chi^2$ to be 10.2 for 9 degrees of freedom. The $\chi^2$ probability is 34%.

We derived the $|V_{ub}|$ values from the partial or total decay rates, using different theoretical calculations:

- for partial $\Delta BR(q^2 > 16\text{ GeV}^2)$ we used the unquenched lattice QCD calculations by HPQCD [25] and FNAL [23], and the quenched QCD calculation by APE [68].
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The values of $\Delta \zeta$ and their uncertainties have been supplied by the authors of [20, 25, 23, 68]. We then calculated $|V_{ub}|$ values using the Eq.7.9.

Tab. 7-7 summarizes the values of $|V_{ub}|$ extracted from the partial and total branching fraction measurements with the tagged $B$ mesons.

<table>
<thead>
<tr>
<th>$q^2 &lt; 8\text{ GeV}^2$</th>
<th>$8 &lt; q^2 &lt; 16\text{ GeV}^2$</th>
<th>$q^2 &gt; 16\text{ GeV}^2$</th>
<th>All $q^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0$</td>
<td>$0.383 \pm 0.121 \pm 0.036$</td>
<td>$0.333 \pm 0.101 \pm 0.030$</td>
<td>$0.474 \pm 0.129 \pm 0.059$</td>
</tr>
<tr>
<td>$B^+$</td>
<td>$0.176 \pm 0.066 \pm 0.018$</td>
<td>$0.429 \pm 0.102 \pm 0.043$</td>
<td>$0.218 \pm 0.091 \pm 0.047$</td>
</tr>
<tr>
<td>Combined</td>
<td>$0.355 \pm 0.086 \pm 0.029$</td>
<td>$0.518 \pm 0.097 \pm 0.041$</td>
<td>$0.457 \pm 0.104 \pm 0.056$</td>
</tr>
</tbody>
</table>
Branching Fractions and $|V_{ub}|$ extraction

### FF model $q^2$ range $\Delta \zeta$ (ps$^{-1}$) $|V_{ub}|$ (10$^{-3}$)

| FF model | $q^2$ range | $\Delta \zeta$ (ps$^{-1}$) | $|V_{ub}|$ (10$^{-3}$) |
|-----------|-------------|-----------------|------------------|
| Ball04    | $< 16$ GeV$^2$ | 5.44±1.43       | 3.23 ± 0.24 ± 0.12 +0.53 −0.36 |
| HPQCD     | $> 16$ GeV$^2$ | 1.46±0.35       | 4.51 ± 0.51 ± 0.28 +0.66 −0.46 |
| FNAL      | $> 16$ GeV$^2$ | 1.83±0.50       | 4.03 ± 0.46 ± 0.25 +0.70 −0.46 |
| APE       | $> 16$ GeV$^2$ | 1.80±0.86       | 4.07 ± 0.46 ± 0.25 +1.50 −0.72 |
| Ball04    | Total         | 7.74±2.32       | 3.35 ± 0.21 ± 0.13 +0.65 −0.41 |
| HPQCD     | Total         | 5.70±1.71       | 3.90 ± 0.24 ± 0.16 +0.76 −0.48 |
| FNAL      | Total         | 6.24±2.12       | 3.73 ± 0.23 ± 0.15 +0.80 −0.51 |
| APE       | Total         | 7.0±2.9         | 3.52 ± 0.22 ± 0.14 +1.08 −0.56 |

Table 7-7. Values of $|V_{ub}|$ derived from combined semileptonic- and hadronic-tag measurements.

charged and neutral $B$ mesons. Presently the untagged methods still give the best experimental precision for the branching fraction, since the tagged techniques are still statistically limited. Anyway, the total branching fraction obtained from the combination of the results coming from the hadronic and semileptonic tags, $BR(B^+ \rightarrow \pi^0 \ell^+ \nu) = (1.33 \pm 0.17_{stat} \pm 0.11_{syst}) \cdot 10^{-4}$, has the smallest systematic uncertainty among the existing measurements thanks to the superior signal purity and the overall precision is comparable to the best.

### Experiment - Tag | Mode | $\overline{B}B$ (fb$^{-1}$) | Branching Fraction (10$^{-4}$)

<table>
<thead>
<tr>
<th>Experiment - Tag</th>
<th>Mode</th>
<th>$\overline{B}B$ (fb$^{-1}$)</th>
<th>Branching Fraction (10$^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO [40]</td>
<td>U</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>9.1</td>
</tr>
<tr>
<td>BABAR [42]</td>
<td>U</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>206</td>
</tr>
<tr>
<td>Belle [44]</td>
<td>S</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>253</td>
</tr>
<tr>
<td>Belle [44]</td>
<td>S</td>
<td>$B^+ \rightarrow \pi^0 \ell \nu$</td>
<td>253</td>
</tr>
<tr>
<td>BABAR [43]</td>
<td>S</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>211</td>
</tr>
<tr>
<td>BABAR [43]</td>
<td>S</td>
<td>$B^+ \rightarrow \pi^0 \ell \nu$</td>
<td>211</td>
</tr>
<tr>
<td>BABAR [43]</td>
<td>F</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>211</td>
</tr>
<tr>
<td>BABAR [43]</td>
<td>F</td>
<td>$B^+ \rightarrow \pi^0 \ell \nu$</td>
<td>211</td>
</tr>
<tr>
<td>Belle [69]</td>
<td>F</td>
<td>$B^0 \rightarrow \pi^- \ell \nu$</td>
<td>252</td>
</tr>
<tr>
<td>Belle [69]</td>
<td>F</td>
<td>$B^+ \rightarrow \pi^0 \ell \nu$</td>
<td>252</td>
</tr>
</tbody>
</table>

Table 7-8. Summary of the recent measurements of branching fractions of exclusive charmless semileptonic $B \rightarrow \pi \ell \nu$ decay. In each case, the first error is statistical, the second is systematic. U indicates untagged method, S semileptonic tagging method, and F full reconstruction tagging.

Most of the analysis listed in Tab.7-8 have produced values for $|V_{ub}|$, based either on partial branching fraction values over the limited $q^2$ region for which selected theoretical predictions for the form factor normalizations are applicable, or by using the full $q^2$ region and an extrapolation of the predictions to the full

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region. In general the precision of the extracted $|V_{ub}|$ values is dominated by the form factor uncertainties, with the overall precision on $|V_{ub}|$ tending to be better when a limited $q^2$ range is employed.

The situation can be summarized by quoting the approach taken by HFAG [38]. The $B \to \pi \ell \nu$ branching fraction measurements from all the analysis are combined to give a global average in three $q^2$ ranges: the full range which extends to approximately $25$ GeV/$c^2$, and the ranges $q^2 < 16$ GeV/$c^2$, to which LCSR applies, and $q^2 > 16$ GeV/$c^2$, to which LQCD applies. $|V_{ub}|$ is then calculated using the theoretical predictions and their stated theoretical errors. The resulting $|V_{ub}|$ values are displayed in Fig. 7-4. The agreement among the existing measurements is satisfactory. The differences among the available form-factor calculations remain considerable, although they are consistent with the quoted theoretical uncertainties.

At this point in time a global $|V_{ub}|$ average based on exclusive decays is not quoted by HFAG, so an overall uncertainty on the exclusive $|V_{ub}|$ is difficult to quote. Anyway, considering only the experimental errors, the actual measurements of $B \to \pi \ell \nu$ partial branching fractions can determine $|V_{ub}|$ with a precision of about $\pm 5\%$.

Using the hadronic tag technique, we also measured the branching fraction of $B^+ \to \eta \ell^+ \nu$ decay mode, obtaining a value that is consistent with the measurement performed by CLEO [40]. Moreover, the recoil technique applied to this exclusive analysis has allowed for an estimation of the upper limit for the channel $B^+ \to \eta' \ell^+ \nu$, for which only a previous upper limit measurement from BABAR was available. A summary of existing measurements on the $B \to X_u \ell \nu$ decays other than $B \to \pi \ell \nu$ are given in Tab.7-9.

**BRANCHING FRACTIONS AND $|V_{ub}|$ EXTRACTION**
Although at this time the determination of $|V_{ub}|$ from the other exclusive charmless semileptonic decays looks less promising than for $B \to \pi \ell \nu$, an extensive study with independent measurements of the charmless semileptonic decay modes other than $B \to \pi \ell \nu$ could be important to further constrain the theoretical models and reduce the statistical and systematic uncertainties.

Infact, the $B^+ \to \eta \ell^+ \nu$ mode may provide a valuable cross-check for $B \to \pi \ell \nu$ decay mode in the future. $B \to \rho \ell \nu$ mode is more problematic for high precision: the broad width introduces both experimental and theoretical difficulties, but the $B \to \omega \ell \nu$ mode may provide a more tractable alternative to the $B \to \rho \ell \nu$ mode because of relative narrowness of the $\omega$ resonance. Agreement between accurate $|V_{ub}|$ determinations from $B \to \pi \ell \nu$ and from $B \to \omega \ell \nu$ could add confidence in both. Anyway, much progress in LQCD will be necessary before these channels can be used to extract $|V_{ub}|$ reliably.

Even if at the moment the measurements using the recoil technique are still statistically limited, the approach presented in this thesis looks promising in the long term, when larger samples of $B \bar{B}$ events will be available at the $B$ -Factories. Considering, infact, that the large acceptance of this method permits to select nearly the whole phase space, resulting in a reduction of the uncertainties coming from the theory, with a larger sample of data also the statistical errors would be reduced and a more precise determination of the branching fractions and an exclusive $|V_{ub}|$ measurements competitive with the inclusive one would be possible. Coupling the recoil technique with improvements in lattice calculations, an overall precision of the $|V_{ub}|$ exclusive measurement at $\pm 5\%$ will be possible in the next future.

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### Table 7-9. Summary of the available measurements of branching fractions of exclusive charmless semileptonic decays other than $B \to \pi \ell \nu$. In each case, the first error is statistical, the second is systematic. Where a third error is quoted it corresponds to uncertainties from form factor shape. $U$ indicates untagged method, $S$ semileptonic tagging method, and $F$ full reconstruction tagging.

<table>
<thead>
<tr>
<th>Experiment - Tag</th>
<th>Mode</th>
<th>$\overline{B}B$ ($fb^{-1}$)</th>
<th>Branching Fraction ($10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO [40] U</td>
<td>$B \to \eta \ell \nu$</td>
<td>9.1</td>
<td>$0.84 \pm 0.31 \pm 0.16 \pm 0.09$</td>
</tr>
<tr>
<td>BABAR [70] H</td>
<td>$B^+ \to \eta' \ell^+ \nu$</td>
<td>315</td>
<td>$0.84 \pm 0.27 \pm 0.21$</td>
</tr>
<tr>
<td>BABAR [70] H</td>
<td>$B^+ \to \eta' \ell'^+ \nu$</td>
<td>315</td>
<td>$&lt; 1.3$</td>
</tr>
<tr>
<td>BABAR [70] H</td>
<td>$B^+ \to \eta \ell^+ \nu$</td>
<td>81</td>
<td>$&lt; 4.5$</td>
</tr>
<tr>
<td>CLEO [71] U</td>
<td>$B^0 \to \rho^- \ell \nu$</td>
<td>3.1</td>
<td>$2.69 \pm 0.41^{+0.35}_{-0.40} \pm 0.5$</td>
</tr>
<tr>
<td>BABAR [72] H</td>
<td>$B^0 \to \rho^- \ell \nu$</td>
<td>81</td>
<td>$2.57 \pm 0.52 \pm 0.59$</td>
</tr>
<tr>
<td>BABAR [41] U</td>
<td>$B^0 \to \rho^- \ell \nu$</td>
<td>81</td>
<td>$2.14 \pm 0.21 \pm 0.51 \pm 0.28$</td>
</tr>
<tr>
<td>Belle [44] S</td>
<td>$B^0 \to \rho^- \ell \nu$</td>
<td>252</td>
<td>$2.17 \pm 0.54 \pm 0.31 \pm 0.08$</td>
</tr>
<tr>
<td>Belle [44] S</td>
<td>$B^+ \to \rho^0 \ell^+ \nu$</td>
<td>252</td>
<td>$1.33 \pm 0.23 \pm 0.17 \pm 0.05$</td>
</tr>
<tr>
<td>Belle [74] U</td>
<td>$B \to \omega \ell \nu$</td>
<td>78</td>
<td>$1.3 \pm 0.4 \pm 0.2 \pm 0.3$</td>
</tr>
<tr>
<td>BABAR [72] H</td>
<td>$B \to \omega \ell \nu$</td>
<td>81</td>
<td>$1.26 \pm 0.55 \pm 0.24$</td>
</tr>
</tbody>
</table>
Conclusions

In this thesis we present the study of the exclusive charmless semileptonic decays $B \rightarrow X_u \ell \nu$ ($X_u = \pi, \pi^0, \eta, \eta'$). These decays provide a clean environment to study $b \rightarrow u$ transitions and to determine the $V_{ub}$ parameter of the $CKM$ matrix.

The analysis is based on a sample of $B \bar{B}$ events produced at the $\Upsilon(4S)$ resonance, in which the hadronic decay of one $B$ meson is fully reconstructed and the exclusive charmless semileptonic decays of the recoiling $B$ meson are studied. This approach assures an high purity of the signal and a low level of background, therefore it is possible to apply very loose kinematic cuts and to analyze nearly the whole phase space. Thus, the theoretical uncertainties due to the form factors and the variations of the form factors as function of $q^2$ are highly reduced. Since the efficiency of the fully reconstruction of B meson is low ($\sim 10^{-3}$), the statistical significance of this method is still limited. On the other hand theoretical and experimental systematic uncertainties are small.

The focus of the analysis is primarily on $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ decay modes because, among the other exclusive channels, the $\pi \ell \nu$ decay offers the cleanest path to $|V_{ub}|$ both experimentally and theoretically.

The measurement of the partial branching fraction of $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ in ranges of $q^2$ on a data sample of 211fb$^{-1}$ has been performed. Using isospin symmetry, the measurements have been combined to express the results as the $B^0$ branching fraction and they are reported in Tab.7-3. The measured total branching fraction is $BR(B^0 \rightarrow \pi^- \ell^+\nu) = 1.23 \pm 0.25_{stat} \pm 0.13_{syst} \cdot 10^{-4}$ Then the current theoretical predictions of the form factors have been used to determine the magnitude of $|V_{ub}|$, ranging between $2.8 \cdot 10^{-3}$ and $5.4 \cdot 10^{-3}$ (detailed results in Tab.7-4). As an example, the recently published unquenched lattice QCD calculations [23] gives $|V_{ub}| = 3.6 \pm 0.4_{stat} \pm 0.2_{syst} \pm 0.8_{0,F_F} \cdot 10^{-3}$. 

These results have been combined with the measurements obtained by $BABAR$ using the semileptonic tag technique [43]. The measured total branching fraction using the tagged technique, $BR(B^0 \rightarrow \pi^- \ell^+\nu) = 1.33 \pm 0.17_{stat} \pm 0.11_{syst} \cdot 10^{-4}$, has the smallest systematic uncertainty among the existing measurements [40, 41, 44] thanks to the superior signal purity, and the overall precision is comparable to the best. Using theoretical calculation of the form factors, we obtain values of $|V_{ub}|$, ranging between $3.2 \cdot 10^{-3}$ and $4.5 \cdot 10^{-3}$ (Tab. 7-7).

The current measurements of $|V_{ub}|$ from $B \rightarrow \pi \ell \nu$ decays has reached an experimental precision of about $5\%$ for the full $q^2$ range, but the theoretical uncertainties are dominant. The approach presented in this thesis looks the most promising, since the large acceptance of this method permits to select nearly the whole phase space, resulting in a reduction of the uncertainties coming from the theory.
Progress in the determination of $|V_{ub}|$ depends critically also on further improvement in form-factor calculations. A reduction of the theoretical uncertainty to the $5 - 6\%$ level may be feasible in the next few years, which would allow an exclusive determination of $|V_{ub}|$ with a precision of better than $8\%$.

We also performed the measurements of the exclusive semileptonic branching fraction of the $B^+ \to \eta \ell \nu$ and $B^+ \to \eta' \ell \nu$ decays on a data sample of $316 fb^{-1}$. Since the yields for the $B^+ \to \eta \ell^+ \nu$ and $B^+ \to \eta' \ell^+ \nu$ decay modes are not statistically significant, we set un upper limit for these branching fractions:

\begin{align}
B\mathcal{R}(B^+ \to \eta \ell^+ \nu) &< 1.4 \times 10^{-4} (90\% C.L.) \quad (7.11) \\
B\mathcal{R}(B^+ \to \eta' \ell^+ \nu) &< 1.3 \times 10^{-4} (90\% C.L.) \quad (7.12)
\end{align}
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